Growth Options, Incentives, and Pay-for-Performance: Theory and Evidence *

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Abstract

When firm value is non-linear in manager effort, pay-for-performance, measured as the sensitivity of manager compensation to firm value, is not a sufficient statistic for the strength of managerial incentives. To demonstrate this effect, we characterize the optimal contract between an investor and a risk-averse manager in the presence of a lumpy investment option. In our model, increasing the size of the growth option can decrease pay-performance sensitivity despite always increasing managerial effort and incentives. Low pay-performance sensitivity is consistent with higher effort and incentives because increasing the size of the growth opportunity increases the sensitivity of firm value to managerial effort. We document new empirical evidence consistent with our model. In a within firm analysis, a one standard deviation increase in Market-to-Book, a proxy for the presence of growth options, is associated with a roughly 6.5% decrease in Jensen and Murphy’s (1990) pay-performance sensitivity, as measured by dollar changes in manager wealth to dollar changes in firm value.

Keywords: Dynamic Contracting, Real Options, Pay-Performance Sensitivity.

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1 Introduction

A fundamental insight from agency theory is that incentives require pay for performance (Jensen and Meckling, 1976). In the context of a firm subject to agency conflicts, this insight means that if shareholders wish to motivate managers to take actions that increase firm value, then manager pay must depend on firm performance. Given the strength of the theoretical relationship between incentives and pay for performance, a large literature has developed that estimates managerial incentives by measuring pay-performance sensitivity (PPS) (Baker and Hall, 2004; Jensen and Murphy, 1990a; Murphy, 1985). A key finding in this literature is that PPS is remarkably low (Frydman and Jenter, 2010). This has lead many researchers to conclude that managers have weak incentives.

We argue that the relationship between PPS and incentives is confounded by the unobserved sensitivity of firm performance to managerial effort. Differences in this sensitivity between firms and over time within a given firm result in differing incentives even in the absence of measurable differences in PPS. We identify growth options as an important channel that drives within firm variation in the sensitivity of firm performance to effort, in that this sensitivity increases as the size of growth options increases.

To formalize this argument, we present a model of optimal incentives and real options in the spirit of Gryglewicz and Hartman-Glaser (2016). In our model, a risk-averse manager operates a firm on behalf of an investor. The manager can exert effort to affect the growth rate of productivity of the firm. The investor cannot directly observe the manager’s effort choice, and hence provides incentives by making the manager’s pay contingent on changes in firm value. The manager is risk averse and the investor is risk neutral, so providing these incentives is costly. While the manager controls the productivity of the firm, the investor has a real option to increase the firm’s capital stock.

The value of the firm is the present value of the firm’s assets in place plus the value of the growth option. While manager effort improves the value of the existing assets directly, it also increases the value of the growth option by bringing the firm closer to the optimal exercise
boundary of that option. Standard intuition implies that the value of the growth option is convex in the underlying asset value of the firm. As a result, when the growth option makes up a large share of firm value, a small increase in managerial effort leads to a large increase in firm value. This in turn implies that the manager’s pay need not be highly sensitive to firm value in order for her to have strong incentives to exert effort. Importantly, increasing the size of the growth option can actually decrease pay-performance sensitivity as measured by the sensitivity of dollar changes in the managers wealth with respect to dollar changes in firm value. However, this does not imply the manager’s actual incentives decrease, just that the measure of performance has become more sensitive to her effort.

To solve our model, we first formulate the agent’s incentive compatibility constraint and then use this to characterize the optimal contract. In the optimal contract, the manager’s incentives are provided through sensitivity of the manager’s compensation to unexpected changes to firm productivity. To operationalize this sensitivity, the firm can either implement cash bonuses tied to a benchmark cash flow level, or employ a dynamic stock account in which the amount of stock owned by the manager is dependent on the current productivity of the firm.

In characterizing the manager’s pay performance sensitivity, we observe that managerial incentives can vary without matching variation in the manager’s effort level. As the size of the firm’s growth option increases, firm value becomes more sensitive to manager effort, and reduced PPS is sufficient to incentivize the manager. Because the manager is risk averse, excessive exposure is costly, so that the optimal contract implements this lower PPS. Naturally, the same effect holds in reverse when the size of growth options is reduced. Put differently, effects on pay-for-performance metrics are not necessarily indicative of manager incentives and effort.

We go on to present new evidence on the relationship between pay performance sensitivity and growth options. Using data on pay performance sensitivity calculated by Coles, Daniel, and Naveen (2013), as well as executive and firm characteristics from Execucomp
and Compustat, we find that pay performance sensitivity is negatively related to proxies for growth options, consistent with our model. Specifically, we regress dollar changes in manager wealth to dollar changes in firm value, a measure of pay performance sensitivity suggested by Jensen and Murphy (1990a), on Market-to-Book or R&D expenditures as well as a firm and manager characteristics and industry and year fixed effects. We find that, for a given firm, a one standard deviation increase in Market-to-Book is associated with a roughly 4.9% decrease in pay performance sensitivity.

Our work is related to a large literature on executive compensation. Frydman and Jenter (2010) and Murphy (2013) provide a comprehensive review of the theoretical and empirical findings in this literature. In emphasizing an important aspect in the empirical evaluation of pay-performance sensitivity, we are indebted to the prior literature that extensively documented the importance of managerial incentives in firm decision making. Coles, Daniel, and Naveen (2006); Hirshleifer and Suh (1992) and Rajgopal and Shevlin (2002) are among the many papers that document the effect of managerial incentives on operational decisions. There has also been work on the effect of incentives on other financing decisions, as studied by Babenko (2009) and Chava and Purnanandam (2010), among others.

Theoretical work has focused on characterizing the optimal contract, both in terms of its exposure and level. In the model of Gabaix and Landier (2008), the equilibrium is one in which larger firms are those with the most talented managers, who command higher wages. In our model, compensation is tied to firm output, but the growth option makes firm value a non-linear function of output, and so, while compensation is increasing in firm size, the underlying dynamics differ. Also of note are the models of Chaigneau, Edmans, and Gottlieb (2014) and Edmans and Gabaix (2011), which characterize optimal managerial contracts under slightly different settings. An important observation made by Baker and Hall (2004) is that the optimal structure of compensation depends on the assumption of how managerial effort impacts firm value.

As in the models of Lambert (1983); Rogerson (1985) and Edmans, Gabaix, Sadzik,
and Sannikov (2012), a feature of the optimal contract is the effect of present performance on both current and future compensation. While those models are in discrete time, our work is like that of DeMarzo and Sannikov (2006) and He, Li, Wei, and Yu (2013) in that it employs a continuous-time setting. A continuous-time model is desirable because it permits characterization of the optimal contract and the firm’s value function using ordinary differential equations.

In our model, real options are a source of convexity in firm payoffs. First introduced in Brennan and Schwartz (1985), there has been a substantial literature analyzing the presence and implications of investment opportunities as options. Berk, Green, and Naik (1999) finds that the optimal exercise of investment opportunities can simultaneously reproduce a multitude of cross-sectional asset pricing features. Carlson, Fisher, and Giammarino (2004) builds on this analysis by introducing operating leverage and reversible investment. In a similar spirit, by analyzing real options in the context of managerial incentives, we work to understand the rich interdependence between managerial decision making and investment opportunities.

By studying the effect of real options on incentives, our paper contributes to the literature on manager incentives. The seminal paper in this area is Holmstrom, Milgrom, et al. (1987), which studies the contract between a risk-averse manager and a risk-neutral firm. Our model is similar to that of He (2011) in that it features a risk-averse manager who can exert effort to increase expected cash flows. Unlike that model, our model gives the firm a growth option. Similar to earlier models, there are two kinds of costs in implementing effort, as described first in Holmstrom et al. (1987), the direct monetary cost, and the risk-compensation term for the risk-averse agent to bear incentives.

Empirical work on the measurement of PPS was pioneered by the competing measures of Jensen and Murphy (1990b) and Hall and Liebman (1998). An important contribution was made by Core and Guay (2002) in providing a methodology for estimating the sensitivity of option-based compensation. Our work both relies upon and contributes to the measurement
of PPS by identifying growth options as an important source of variation in PPS. In this way, our work contributes to the literature on the determinants of executive compensation (Baker, Jensen, and Murphy, 1988; Deckop, 1988; Yermack, 1995; Becker, 2006).

Despite our emphasis on PPS, which is typically measured for public firms, our model is readily applicable to the example of private equity portfolio companies, where the aggressive use of financial leverage introduces convexity into the investor’s value function. Our characterization of the optimal contract is consistent with the empirical findings of Cronqvist and Fahlenbrach (2013), who find that principals at private equity firms tie CEO bonuses to cash-flow benchmarks. The investor provides incentives tied to unexpected shocks in the firm’s cash flows. In our model, managerial effort directly impacts the growth of cash flows, so that cash-flow sensitivity is a direct measure of her incentives. However, the provision of incentives through stock accounts and the non-linear relationship between cash flow and firm value mean that observed declines in manager stock ownership cannot be interpreted as decreased managerial incentives nor decreased managerial effort.

Finally, our paper is related to the literature on option exercise in the presence of agency problems. Grenadier and Malenko (2011) and Kruse and Strack (2015) characterize optimal contracts in this setting. The set up of our model follows that of Gryglewicz and Hartman-Glaser (2016), which looks at the timing of investment decisions in the presence of agency conflicts. Rather than focus on the investment decision, we focus on the problem of how growth options can impact manager incentives.

2 The model

Time is continuous and indexed by $t$. An infinitely-lived firm generates a continuous cash flow given by $X_t K_t$, where $K_t$ is the capital base and $X_t$ is firm productivity. Capital $K_t$ takes initial value $K_0 = k_s$ and the firm has a real option to pay $P$ and increase capital to $k_b$. Let $\tau$ be the time of investment.
A risk-neutral investor hires a risk-averse manager to run the firm. The common discount rate is denoted by $r$. Both $X_t$ and $K_t$ are observable to the investor, and thus the manager has no ability to divert cash flows. A moral hazard problem arises because the manager affects the firm’s productivity. Specifically, prior to investment, productivity $X_t$ depends on manager effort $a_t \in [0, 1]$ and follows the process

$$dX_t = a_t \mu X_t \, dt + \sigma X_t \, dZ_t,$$

where $\mu$ and $\sigma$ are positive constants. We assume that $\mu < r$ for the problem to have a finite solution. $dZ_t$ is a Brownian motion that is unobservable to the investor. In such an environment, the manager’s effort is also unobservable to the investor.

In our model, the value of the manager is due to her ability to grow the firm’s productivity $X_t$. This view of a manager is consistent with characterizations of CEOs as focused on firm growth and future performance. As our interest lays in the interaction of agency conflicts and growth opportunities, we simplify the analysis and assume that after investment at time $\tau$, firm productivity stays at $X_\tau$ forever and there are no agency conflicts. Thus the post-investment value of the firm is simply $(X_\tau k_b)/r$. In what follows, we focus on optimal contracting and valuation of the firm prior to investment.

The investor receives cash flows from the firm and pays the manager compensation $c_t$, so that his net cash flow $D_t$ follows dynamics given by

$$dD_t = X_t K_t \, dt - c_t \, dt - P \, 1(t = \tau).$$

The manager values a stream of consumption and effort $\{c_t, a_t\}$ as

$$U(\{c_t, a_t\}) = E \left[ \int_0^\infty e^{-rs} u(c_t, a_t) \, dt \bigg| \{a_t\} \right].$$

where her instantaneous utility is

$$u(c_t, a_t) = -\frac{1}{\gamma} \exp \{-\gamma [c_t - g(a_t)X_t] \}.$$  \hspace{1cm} (4)

The manager’s private cost of effort $X_tg(a_t)$ is measured in units of consumption. $g(a)$ is assumed to be continuous, increasing and convex in effort $a$: $g(a) \in C^1([0, 1])$, $g'(a) > 0$, $g''(a) > 0$, $g'(0) = 0$, and $g'(1) = \infty$. Such effort costs ensure that any optimal contract will specify interior effort in $(0, 1)$. The cost of effort is increasing in the firm’s current level of productivity, and therefore in firm size. This captures the intuition that it is more difficult and costly for the manager to improve productivity of an already productive firm.

The manager chooses an effort level $a_t \in [0, 1]$ and has the ability to engage in unobserved savings at rate $r$. Furthermore, the manager has a competing offer of $\{\tau_t, \bar{a}_t\}$, which she values at $w_0$.

A contract consists of a compensation rule, a recommended effort level, and an investment policy, denoted $\Pi = (\{c_t, a_t\}, \tau)$, where $\{c_t\}$ and $\{a_t\}$ are stochastic processes adapted to the filtration of public information $\mathcal{F}_t$ and $\tau$ is an $\mathcal{F}_t$-stopping time.

Given contract $\Pi$ and initial productivity $X_0$, the manager chooses stochastic processes $\{\tilde{c}_t\}$ and $\{\tilde{a}_t\}$ to maximize her utility from the contract:

$$W(\Pi) = \max_{\tilde{c}, \tilde{a}} \mathbb{E} \left[ \int_0^\infty -\frac{1}{\gamma} \exp \{-\gamma [\tilde{c}_t + g(\tilde{a}_t, X_t)X_t] \} - rt \right] | X_0 = X_0$$

such that

$$dS_t = rS_t \, dt + (c_t - \tilde{c}_t) \, dt, \text{ where } S_0 = 0$$

$$dX_t = \tilde{a}_t \mu X_t \, dt + \sigma X_t \, dZ_t$$

$$K_t = k_s + (k_b - k_s) \mathbb{I}(t \geq \tau).$$ \hspace{1cm} (5)
The investor’s value of contract \( \Pi \) is:

\[
v(\Pi) = \mathbb{E}_{\{\tilde{a}_t\}} \left[ \int_0^\infty e^{-rt} dD_t \mid X_0 = X_0 \right]
\]

where \( dX_t = \tilde{a}_t \mu X_t \, dt + \sigma X_t \, dZ_t \)

\[
K_t = k_s
\]

\( \{\tilde{c}_t, \tilde{a}_t\} \) solves (5).

Therefore, the investor chooses the optimal contract to solve:

\[
\Pi^* (X_0, w_0) = \max_{\Pi} v(\Pi)
\]

such that \( W(\Pi) \geq w_0 \).

A contract \( \Pi \) is termed *incentive-compatible* and *zero-savings* if the manager’s choice of \( \{\tilde{c}_t, \tilde{a}_t\} \) is equal to the payment rule and recommended effort plan \( \{c_t, a_t\} \) given in \( \Pi \). We restrict our attention to such contracts by virtue of the following version of the revelation principle.

**Lemma 1.** Let \( \bar{\Pi} \) be a contract and \( X_0 \) be the firm’s initial productivity. There exists an incentive-compatible and zero-savings contract \( \Pi \) that satisfies \( v(\Pi) \geq v(\bar{\Pi}) \) and \( W(\Pi) \geq W(\bar{\Pi}) \).

### 2.1 No-Savings and Incentive-Compatibility Conditions

The property that the contract features zero-savings leads almost immediately to implications regarding the optimal wage function. Suppose \( \{\tilde{c}, \tilde{a}\} \) solves the manager’s problem for a given contract \( \Pi \) and implements zero savings. Further suppose that the manager is endowed with savings \( S > 0 \) at time \( t \geq 0 \). Because the manager has CARA preferences, the optimal consumption plan for \( s \geq t \) will be \( \tilde{c}_s + rS \), and her effort provision \( \tilde{a}_s \) would be unchanged. Thus, an increase in savings from 0 to \( S \) increases the manager’s instantaneous
utility by a factor of \( \exp\{ -\gamma rS \} \) for \( s \geq t \). Therefore, we can write the manager’s utility for contracts \( \Pi \) and savings \( S \) as:

\[
W_t^* (\Pi, S) = \exp\{ -\gamma rS \} W_t (\Pi). \tag{8}
\]

In order for the zero-savings condition to hold, it must be the case that

\[
u_c (\tilde{c}_t, \tilde{a}_t) = \frac{\partial}{\partial S} W_t^* (\Pi, 0), \tag{9}\]

which implies that

\[-\gamma u (\tilde{c}_t, \tilde{a}_t) = -\gamma r W_t (\Pi), \tag{10}\]

or

\[u (\tilde{c}_t, \tilde{a}_t) = r W_t (\Pi). \tag{11}\]

**Lemma 2.** A contract implements zero-savings if and only if the manager’s instantaneous utility is equal to the yield on her continuation utility

\[u (c_t, a_t) = r W_t. \tag{12}\]

Next, we characterize the condition for incentive compatibility. By the Law of Iterated Expectations, for an arbitrary incentive-compatible and zero-savings contract, the process

\[M_t = E_t \int_0^\infty e^{-rs} u (c_{t+s}, a_{t+s}) \, ds \tag{13}\]

must be a martingale with respect to filtration \( \mathcal{F}_t \). By the Martingale Representation Theorem, there exists a progressively measurable process \( \beta_t \) such that

\[dM_t = \beta_t (-\gamma r W_t) e^{-rt} (dX_t - a_t \mu X_t dt - \sigma X_t \, dZ_t). \tag{14}\]
Note that $M_t$ is related to the manager’s continuation utility $W_t$ (under the recommended consumption and effort plan) by:

$$dW_t = (rW_t - u(c_t, a_t)) \, dt + e^{r_t}dM_t.$$  \hfill (15)

This gives the following dynamics for the manager’s continuation utility:

$$dW_t = \beta_t (-\gamma r W_t) (dX_t - a_t \mu X_t \, dt)$$  \hfill (16)

The process $\beta_t$ is the sensitivity of the manager’s continuation utility to unexpected shocks to the firms’ productivity. Since a deviation from the recommended effort policy results in a shock to productivity, $\beta_t$ measures the manager’s incentive to deviate from the contract’s recommended effort policy.

Now, consider the manager’s choice of effort $\tilde{a}_t$. Given that the manager chooses her effort level $\tilde{a}_t$ to maximize the sum of her instantaneous utility $u(c_t, \tilde{a}_t) \, dt$ and the expected change in her continuation utility $W_t$, her expected change in continuation utility achieved by a deviation from the recommended effort level $a_t$ to $\tilde{a}_t$ is:

$$\mathbb{E} [dW_t|\tilde{a}] = \beta_t (-\gamma r W_t) (\tilde{a}_t - a_t) \mu X_t \, dt.$$  \hfill (17)

In order for the recommended effort level $a_t$ to be incentive-compatible, it must be the case that:

$$a_t \in \arg \max_{\tilde{a}} \left\{ u(c_t, \tilde{a}_t) + \beta_t (-\gamma r W_t) (\tilde{a}_t - a_t) \mu X_t \right\}.$$  \hfill (18)

By our assumptions about the cost function $g$, the optimal choice of effort will take on an interior solution in the interval $(0, 1)$. Taking first-order condition yields:

$$u_a(c_t, a_t) + \beta_t (-\gamma r W_t) \mu X_t = 0$$  \hfill (19)
Substituting \( u_a(c_t, a_t) = -u_c(c_t, a_t) g'(a_t) X_t \) and the no-savings condition, we can rearrange the first-order condition above:

\[
\beta_t = \frac{g'(a_t)}{\mu}.
\]  

(20)

Intuitively, incentive-compatibility requires that the sensitivity of the manager’s continuation utility to unexpected output shocks \( \beta_t \) be weakly greater than her marginal cost of effort \( g_a(a_t) X_t \), scaled by the marginal impact of effort on output, \( \mu X_t \).

**Lemma 3.** A contract is incentive-compatible and implements zero savings if and only if the solution \( W_t \) to the manager’s problem has dynamics given by (16), where \( \beta_t \) is defined by (20).

As in other dynamic agency models, the agent’s continuation utility \( W_t \) can be used as a state variable to solve for the optimal contract. It is useful to further transform \( W_t \) into its certainty equivalent

\[
Y_t = \frac{1}{\gamma r} \ln(-\gamma r W_t),
\]  

(21)

so that we can take \( Y_t \) to be a state variable for the investor’s problem in place of \( W_t \). Applying Itô’s Lemma yields that the dynamics of \( Y_t \) under an incentive-compatible, zero-savings contract are given by:

\[
dY_t = \frac{1}{2} \gamma r (\sigma \beta_t X_t)^2 \, dt + (\sigma \beta_t X_t) \, dZ_t.
\]  

(22)

Although \( W_t \) is a martingale, the difference in risk aversion between the investor and the manager implies that the certainty equivalent \( Y_t \) must have additional drift for each additional unit of volatility. This positive drift will show up in the investor’s Hamilton-Jacobi-Bellman (HJB) equation as the cost of providing incentives.
2.2 Solving for the optimal contract

We now present a heuristic derivation of the optimal contract. First, we characterize the payment rule to the manager. Recall that the zero-savings condition links the manager’s instantaneous utility \( u(c_t, a_t) \) and her continuation utility \( W_t \). This allows us to express the manager’s compensation as a function of the state of the firm \( X_t \), the recommended effort level \( a_t \), and the certainty equivalent \( Y_t \):

\[
    c_t = rY_t - g(a_t)X_t. \tag{23}
\]

We see that the manager’s total compensation is the yield on her continuation utility, less her private benefit of shirking.

Second, we make a useful observation that simplifies the characterization of the optimal contract. The investor’s value function \( v(X, Y) \) is defined in terms of both the firm’s productivity \( X \) and the manager’s continuation utility \( Y \). Due to the absence of wealth effects implied by the manager’s CARA preferences, maximizing the investor’s payoff is equivalent to maximizing the investor’s value function \( v(X, Y) \) plus the certainty equivalent of the manager’s continuation utility \( Y \). Thus, we maximize the total firm value \( V \):

\[
    V(X) = v(X, Y) + Y. \tag{24}
\]

Note that there is no dependence on \( Y \) in \( V(X) \), as the risk-neutral investor values the manager’s consumption stream at exactly his certainty equivalent.

To determine the optimal recommended effort, we take the dynamic programming approach. Over any interval of time in which there is no investment, Itô’s Lemma gives the following Hamilton-Jacobi-Bellman equation for \( V(X) \):

\[
    rV(X) = \max_{a \in [0,1]} \left\{ Xk_s + g(a)X - \frac{1}{2} \gamma r \left( \frac{\sigma}{\mu} g'(a)X \right)^2 + a \mu XV''(X) + \frac{1}{2} \sigma^2 X^2 V''(X) \right\}. \tag{25}
\]
Given our assumptions on the manager’s effort cost function $g(a)$, the optimal effort will be interior. Such optimal solutions for the manager’s effort, $a^*$, will satisfy the first-order condition and will depend on $X$:

$$g'(a^*(X)) = \frac{\mu V'(X)}{k_s + \gamma r \left( \frac{a}{\mu} \right)^2 g''(a^*(X))X}.$$  \hspace{1cm} (26)

Because firm value will be monotonically increasing in effort, we can characterize the solution by a threshold level of productivity, $\overline{X}$, at which it is optimal for the investor to exercise his real option. Following standard solution methods, we find this threshold using value-matching and smooth-pasting conditions.

$$V(\overline{X}) = \frac{\overline{X}k_b}{r} - P,$$  \hspace{1cm} (27)

$$V'(\overline{X}) = \frac{k_b}{r}.$$  \hspace{1cm} (28)

**Proposition 1.** The optimal contract is given by the solution to (25), (27), and (28).

### 3 Growth options and optimal incentives

In this section, we consider the implications of real options for managerial incentives. Specifically, we examine the effect of the presence and size of growth options on the measurement of pay-performance sensitivity (PPS). While there has been a robust empirical investigation, reviewed by Murphy (1999) and Frydman and Jenter (2010), into the relation between PPS and firm size, there has been less attention paid to the relation between investment and PPS. Our results provide guidance for empirical analysis presented in the following section.

The manager’s compensation and incentives depend on the level of effort stipulated by the optimal contract. Therefore, we begin our inquiry with a discussion of managerial effort. With our assumptions on the manager’s effort cost function $g(a)$, the optimal effort will be interior. For interior solutions of effort $a$, we use the HJB equations (25) to characterize the
optimal effort policy \( a^*(X) \) by the first-order conditions:

\[
g'(a^*(X)) = \frac{\mu V'(X)}{k_s + \gamma r \left( \frac{\sigma}{\mu} \right)^2 g''(a^*(X)) X}.
\] (29)

In the following analysis, we restrict our attention to parameter values such that the maximum \( a^*(X) \) satisfies the second-order condition.\(^1\) Optimal effort is time-varying with productivity \( X_t \), depends on the primitive parameters of the model, and on the presence of growth opportunities.

To analyze the size of growth opportunities in the model, we look at varying levels of the post-investment capital \( k_b \). Keeping the cost of investment \( P \) constant, increased \( k_b \) means that the growth option is larger and more valuable.

As \( k_b \) increases, optimal effort increases:

\[
\frac{da^*(X)}{dk_b} > 0.
\] (30)

We prove this claim in the appendix. The intuition is that growth options increase the sensitivity of firm value to productivity shocks as the firm approaches the investment threshold. When this sensitivity is high, contracting high effort to grow \( X \) is additionally attractive from the investor’s point of view.

To implement the optimal effort level under moral hazard, the manager needs to be appropriately incentivized. To determine how investment opportunities affect the power of incentives, we look at two alternative measures thereof: cash-flow sensitivity and firm-value sensitivity of manager wealth. A direct measure of a manager’s incentives in our model is the sensitivity of her dollar (certainty-equivalent) continuation utility to productivity shocks.\(^2\)

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\(^1\)If the second-order derivative of the objective function with respect to \( a \) is zero (a knife-edge case given its dependence of \( X \)), then the implicit function theorem is not applicable.

\(^2\)Given a performance measure \( Q \), a standard way of measuring PPS is \( \Delta \text{Manager’s Wealth}/\Delta Q \). The continuous-time analog to this measure is \( dY/dQ \) since \( Y \) measures the dollar value of the manager’s wealth.
Prior to investment, the optimal contract sets this quantity to

$$
\beta^*(X) = \frac{g'(a^*(X))}{\mu}.
$$

Note that this expression follows directly from substituting the optimal effort policy $a^*(X)$ into the incentive compatibility condition given by Equation (20).

A standard approach in the empirical literature to the measurement of pay-performance sensitivity is to compute the sensitivity of the manager’s wealth to changes in firm value as first proposed by Jensen and Murphy (1990b) (Yermack, 1996; Bergstresser and Philippon, 2006). This approach is particularly convenient from an empirical point of view as it is based on firm value changes, which are easy to measure. In contrast, an output-based PPS measure must isolate that output process which is most directly attributable to the manager. If firm value is linear in output $X_t$, this simplification is inconsequential as value-based PPS would be equivalent to direct, output-based PPS, such as $\beta$. However, growth options can lead to a non-linear relationship between firm value and output, and thus between firm value and manager effort. Thus incentives, which are the sensitivity of manager pay to manager effort, differs from PPS, which is the sensitivity of manager pay to firm value. The wedge between these two sensitivities is precisely the sensitivity of firm value to manager effort, which growth options cause to vary both cross sectionally and in the time series.

In our model, as in He (2011), the manager’s dollar value-based PPS is equal to the sensitivity of the manager’s dollar continuation value to changes in firm value, $V(X)$. Under the optimal contract, this quantity is given by:

$$
\phi^*(X) = \frac{\beta^*(X)}{V'(X)} = \frac{g'(a^*(X))k_s}{\mu V'(X)}.
$$

Since $dZ \cdot dt = 0$ and $dZ^2 = dt$, we have

$$
\frac{dY}{dQ} = \frac{dY}{dZ} \frac{dZ}{dQ} = \frac{\frac{\sigma_Q}{\mu} g'(a^*(X))X}{\sigma_Q},
$$

where the numerator is the volatility of $Y$ given in Equation (22) and $\sigma_Q$ is the volatility of $Q$. Under the production technology we have specified, a productivity shock maps directly to a shock to period cash-flows.
Note that while $\phi^*(X)$ is closely related to $\beta^*(X)$, it is scaled by the slope of the value function in output $V'(X)$. Thus, the presence of growth options affects $\phi^*(X)$ by changing both $\beta^*(X)$ and $V'(X)$. As we show in the next proposition, the wedge between $\beta^*$ and $\phi^*$ induced by $V'(X)$ can lead the two measures of PPS to respond in opposite ways to changes in the size of growth options.

**Proposition 2.** Output-based incentives for the manager always increase in the size of growth options measured by $k_b$. Value-based incentives decrease in the size of growth opportunities if the cost of effort is increasingly convex, $g''''(a) > 0$.

Proposition 2 shows that the behavior of output-based incentives is uncomplicated. Larger growth opportunities increase the benefits that the investor derives from managerial effort. To induce this increased effort, output-based incentives increase. However, incentives measured against firm value can decrease in the size of growth opportunities. To establish an intuition for this result, suppose first that the manager is required to exert the same amount of effort with increased growth opportunities. Larger growth opportunities make firm value more sensitive to managerial effort and thus the manager needs less exposure to firm value to be incentivized to exert the same amount of effort. In equilibrium, optimal effort increases with increased growth opportunities. Proposition 2 shows that value-based incentives still decrease if the increase of required effort in response to higher growth opportunities is not too steep, that is, if the cost of effort is increasingly convex.\(^3\)

### 4 Empirical Findings

In this section we provide evidence that value-based pay-performance sensitivity decreases in the size of growth opportunities.

\(^3\)Obviously, if the marginal cost of effort was sufficiently low, then larger growth opportunities could induce high enough increase of effort to call for more exposure to firm value even if the firm value was more sensitive to effort.
4.1 Data

We merge data from three main sources. We use data on pay-performance sensitivity for the period of 1992-2015 at the manager-firm level from the website of Lalitha Naveen. An empirical equivalent of our model’s value-based pay-performance sensitivity is Jensen and Murphy (1990a)’s measure of pay-performance sensitivity, that is, dollar changes in manager wealth divided by dollar changes in firm value. We call this variable Jensen and Murphy PPS and we use the logarithm of it as the dependent variable in the regressions of this section. We merge the PPS data with data on manager characteristics from Execucomp and data on firm characteristics from Compustat for the same period.

We use several proxies for growth opportunities. As there is no consensus in the literature on the measurement of growth opportunities, our approach is to use a broad set of diverse proxies that have been suggested in previous studies. Our first proxy for growth opportunities is market-to-book ratio. Market value is defined as the market value of equity plus the book value of debt, divided by total assets. A number of studies, including Gompers (1995), Collins and Kothari (1989), Korteweg and Polson (2009), and He et al. (2013), have used the market-to-book ratio as a proxy for growth options, and previous theoretic work by Berk et al. (1999) and Carlson et al. (2004) establishes the link between growth options and market-to-book. The use of price data in our proxies is both a blessing and a curse. It is grounded in the assumption that the market incorporates a firm’s future investment opportunities into its stock price, thus elevating the market value of a firm’s assets beyond the book value of those assets. However, as discussed in Berk (1995), the potential for mispricing makes it unsatisfactory to rely solely on this measure. Equally worrying, a relation found using price-based measures can be unrelated to the operating characteristics of the firms, and can instead reflect changes in market risk premia. Despite these well-founded concerns, previous research by Adam and Goyal (2008) and Kallapur and Trombley (1999) has found that

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4Available at http://astro.temple.edu/~lnaveen/data.html. See Core and Guay (2002) and Coles et al. (2006) for the first papers to use these data, as well as Coles et al. (2013) for an explanation of their construction.
market-to-book performs well as a proxy of growth options and investment opportunities. Nevertheless, we also include a number of non-price based growth option proxies.

Our second proxy is value-to-book, as introduced in Rhodes-Kropf, Robinson, and Viswanathan (2005), where value is an regressed estimate of the firm’s true (unobserved) value of equity. This measure attempts to preserve the intuition behind the market-to-book ratio while correcting for potential mispricings, and the decomposition has been used in a number of studies, including, among others, Dong, Hirshleifer, Richardson, and Teoh (2006), Polk and Sapienza (2009), and DeAngelo, DeAngelo, and Stulz (2010). Whereas Rhodes-Kropf et al. (2005) decomposes the market-to-book ratio into three terms: (i) firm-specific mispricing, (ii) industry mispricing, and (iii) value-to-book, we use the two term decomposition found in Lyandres and Zhdanov (2013): (i) firm-specific, within-industry mispricing and (ii) value-to-book. Value for firm $i$ in industry $j$ at time $t$ is estimated by performing a within-industry $j$ regression in logarithms of market value $M$ on book value $B$.

$$\log M_{ijt} = \alpha_{jt} + \beta_{jt} \log B_{ijt} + \varepsilon_{jt}$$

Subtracting log book value from the fitted value from the regression $\hat{M}_{ijt}$ yields an estimate of log value-to-book. As discussed in Rhodes-Kropf et al. (2005), the link between firm value, corrected for mispricings, and book value rests on two assumptions: one linking future returns on equity to future discount rates within industries, and another that book equity grows at a constant rate. To the extent that these assumptions are unsatisfactory, the value-to-book ratio we use will be an imperfect proxy.

Our third proxy for growth opportunities, we use the augmented measure of Tobin’s $q$ constructed by Peters and Taylor (2016). There is a long theoretical literature which states that a markup in the value of a firm’s assets in place over the replacement value of those assets indicates the presence of investment opportunities. However, standard measures of Tobin’s $q$ fail to account for intangible capital, which per accounting rules is usually expensed
rather than capitalized, and thus not found on the firm’s balance sheet. The Peters-Taylor measure attempts to account for intangible capital in calculating the firm’s $q$ measure, and finds that their augmented measure better predicts investment than previous measures.

For our fourth proxy for growth opportunities, we use an investment-based measure found in Kallapur and Trombley (1999) and Lyandres and Zhdanov (2013), R&D scaled by book asset value. This measure is independent of a firm’s price data and is thus uncontaminated by mispricing. The downside is that industry-specific accounting practices restrict the classification of R&D expense, exposing this measure to concerns of a systematic bias that varies by industry. A firm’s growth opportunities may include acquisition opportunities or investments in subsidiaries, which are not included in R&D expense. Kallapur and Trombley (1999) finds that R&D spending is inconsistently correlated with realized measures of realized growth in a three to five year horizon, making R&D-based measures a weaker proxy for short term investment opportunities than market-to-book ratio, which they find to be a more relevant proxy.

Our last set of proxies are based on capital expenditures, and are drawn from Purnanandam and Rajan (2016). The capital expenditure of a firm documents the exercise of growth options and their conversion into physical assets. Like R&D, capital expenditure based measures are independent of the firm’s stock price. To account for the fact that a firm’s capital expenditure includes maintenance costs for an existing capital base, we calculate our first measure as the residual of a regression of firm CapEx scaled by assets, including a firm fixed effect to capture the predictable investment level of the firm. In terms of regression coefficients, this produces identical estimates to a regression in which capital expenditures is directly used as a regressor. The second measure is the residual from a one-lag auto-regressive model of expected scaled capital expenditures, and is thus a better measure of unanticipated capital expenditures.

These proxies are motivated by the fact that a firm’s reported capital expenditures may reflect preexisting projects or other ongoing commitments, making the level of capital ex-
Table 1: Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jensen &amp; Murphy PPS</td>
<td>182,395</td>
<td>1.07</td>
<td>2.646</td>
<td>0.002</td>
<td>18.858</td>
<td>0.285</td>
</tr>
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<td>Market-to-Book</td>
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<td>1.293</td>
<td>0.771</td>
<td>8.529</td>
<td>1.473</td>
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<td>Value-to-Book</td>
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<td>0.566</td>
<td>0.956</td>
<td>4.023</td>
<td>1.636</td>
</tr>
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<td>R&amp;D</td>
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<tr>
<td>Tobin’s Q</td>
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<td>0.044</td>
<td>7.899</td>
<td>0.851</td>
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<td>0.054</td>
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<td>202.475</td>
<td>1.644</td>
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<td>Firm Age</td>
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<td>Profitability</td>
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<td>Leverage</td>
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<td>0.183</td>
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<td>0.82</td>
<td>0.205</td>
</tr>
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<td>Dividend Paying</td>
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<td>1</td>
</tr>
<tr>
<td>CEO Chair</td>
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<td>0.589</td>
<td>0.492</td>
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<td>1</td>
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<tr>
<td>Fraction of Inside Directors</td>
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</tbody>
</table>

The sample covers all executives and firms in Execucomp from 1992 to 2015 and is merged with Compustat data. Jensen & Murphy PPS is the dollar-to-dollar pay-performance sensitivity.

Expenditures a noisy measurement that misrepresents a firm’s growth opportunities. By taking the residual, we better capture the discretionary or uncommitted portion of a firm’s capital expenditures, which better captures the exercise (and thus reduction) of growth options at the firm. A potential downside of capital expenditure based measures is that the price of capital is affected by economy-wide demand, and thus the firm’s level of capital expenditures is exposed to mispricing at a market- or industry-wide level, albeit in a more indirect way than a measure based on the firm’s own stock price.

Our sample then includes all firm-executive combinations ExecuComp from 1992 to 2015. See Table 1 for summary statics. Note that, for all regressions, independent variables are lagged by one year (as in He, Li, Wei, and Yu (2014)).
4.2 Results

First, we regress Jensen and Murphy PPS on Market-to-book and various controls for manager and firm characteristics. Results for these regressions are in Table 2. The dependent variable is the logarithm of the dollar-to-dollar pay-performance sensitivity (Jensen and Murphy PPS). We construct the fixed effects for industry fixed effect using Fama-French (1997) 48 sectors. The fixed effects in the model in column (3) are in firm-executive pairs. All standard errors are robust and clustered at the firm level. The main effect of interest can be seen in the coefficient on Market-to-Book in Column (3). This coefficient states that a one standard-deviation change in Market-to-book is associated with a roughly 4.9% decrease in Jensen and Murphy’s PPS. While the magnitude of the effect on PPS is smaller than that of other firm characteristics such as firm size, this effect is still quite economically significant.

Second, we regress Jensen and Murphy PPS on the Value-to-Book ratio. Results for these regressions are in Table 3. Other than the alternative measure of growth options, all other controls are identical to those in Table 2. The coefficient in column (3) states that a one standard deviation increase in Value-to-Book is associated with a 1.7% decrease in Jensen and Murphy’s PPS.

As an additional check, we regression Jensen and Murphy PPS on Tobin’s $q$. Results for these regressions are in Table 4. All other controls are identical to those in Table 2. The main effect of interest is the coefficient on Tobin’s $q$ in column (3). This states that a one standard deviation increase in Tobin’s $q$ is associated with a 2.7% decrease in PPS, which is consistent with our previous specification.

Next we regress Jensen and Murphy PPS on R&D and various controls for manager and firm characteristics. Results for these regressions are in Table 5. The dependent variable is the logarithm of the dollar-to-dollar pay-performance sensitivity. All other controls are identical to those in Table 2. Again the main effect of interest is the coefficient on R&D in column (3). This states that a one standard deviation increase in R&D expenses is associated with a 3.8% decrease in Jensen and Murphy’s PPS, which is on the same order of magnitude.
Table 2: Investment opportunities (market-to-book ratio) and pay-performance sensitivity

<table>
<thead>
<tr>
<th>Dependent variable: PPS</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market-to-Book</td>
<td>-0.055**</td>
<td>-0.060**</td>
<td>-0.037**</td>
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<td>(-7.23)</td>
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<td>(-6.28)</td>
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<td>Firm Size</td>
<td>-0.406**</td>
<td>-0.361**</td>
<td>-0.327**</td>
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<tr>
<td></td>
<td>(-46.36)</td>
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<td>(-16.79)</td>
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<td>Firm Age</td>
<td>-0.096**</td>
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</tr>
<tr>
<td></td>
<td>(-4.46)</td>
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</tr>
<tr>
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<td>-0.387**</td>
<td>-0.166</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-3.85)</td>
<td>(-1.43)</td>
<td></td>
</tr>
<tr>
<td>Profitability</td>
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<td>0.022</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.16)</td>
<td>(0.24)</td>
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<tr>
<td></td>
<td>(1.02)</td>
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<td>(5.21)</td>
<td>(5.28)</td>
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<tr>
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<td>(-6.23)</td>
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</tr>
<tr>
<td>CEO Chair</td>
<td>0.116**</td>
<td>0.002</td>
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<tr>
<td></td>
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<td>Fraction of Inside Directors</td>
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<tr>
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<td>CEO</td>
<td>1.749**</td>
<td>0.339**</td>
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<tr>
<td></td>
<td>(86.75)</td>
<td>(20.11)</td>
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<tr>
<td>Female</td>
<td>-0.271**</td>
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</tr>
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<td>(-7.91)</td>
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</tr>
<tr>
<td>Industry Dummies</td>
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<td>Yes</td>
<td>No</td>
</tr>
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<td>Year Dummies</td>
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<td>Yes</td>
<td>Yes</td>
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<tr>
<td>Observations</td>
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</tr>
<tr>
<td>$R^2$</td>
<td>0.276</td>
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</table>

The sample covers all executives and firms in Execucomp from 1992 to 2015 and is merged with Compustat data. The dependent variable is the logarithm of the dollar-to-dollar pay-performance sensitivity. Market value is defined as the market value of equity plus the book value of debt, divided by total assets. Control variables are defined in the caption of Table 1. $t$ statistics based on heteroskedasticity-consistent and firm-level-clustered standard errors are provided in parentheses. Significance levels: * $p < 0.05$, ** $p < 0.01$. 

22
Table 3: Investment opportunities (value-to-book ratio) and pay-performance sensitivity

<table>
<thead>
<tr>
<th>Dependent variable: PPS</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value-to-Book</td>
<td>−0.052*</td>
<td>−0.070**</td>
<td>−0.029*</td>
</tr>
<tr>
<td></td>
<td>(−2.24)</td>
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<td>(−1.99)</td>
</tr>
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<td>Firm Size</td>
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<td>−0.346**</td>
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<td>(−47.40)</td>
<td>(−35.78)</td>
<td>(−17.66)</td>
</tr>
<tr>
<td>Firm Age</td>
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<td>−0.299**</td>
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</tr>
<tr>
<td></td>
<td>(−3.70)</td>
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<tr>
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</tr>
<tr>
<td></td>
<td>(−3.02)</td>
<td>(−1.09)</td>
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<tr>
<td>Profitability</td>
<td>−0.721**</td>
<td>−0.237*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(−5.34)</td>
<td>(−2.51)</td>
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<td></td>
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<td>(−5.17)</td>
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</tr>
<tr>
<td>CEO Chair</td>
<td>0.166**</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(7.96)</td>
<td>(1.78)</td>
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<td>0.685**</td>
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<td>(8.62)</td>
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<td>(−8.85)</td>
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<td>$R^2$</td>
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The sample covers all executives and firms in Execucomp from 1992 to 2015 and is merged with Compustat data. The dependent variable is the logarithm of the dollar-to-dollar pay-performance sensitivity. Value-to-book is calculated as the fitted value from a within-industry regression of log market value on log book value, less log book value. Control variables are defined in the caption of Table 1. $t$ statistics based on heteroskedasticity-consistent and firm-level-clustered standard errors are provided in parentheses. Significance levels: * $p < 0.05$, ** $p < 0.01$. 
Table 4: Investment opportunities (Tobin’s Q) and pay-performance sensitivity

<table>
<thead>
<tr>
<th>Dependent variable: PPS</th>
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<th>(3)</th>
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<tr>
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<td></td>
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<td>(1.43)</td>
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<td>Yes</td>
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<td>$R^2$</td>
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The sample covers all executives and firms in Execucomp from 1992 to 2015 and is merged with Compustat data. The dependent variable is the logarithm of the dollar-to-dollar pay-performance sensitivity. Tobin’s Q is taken from WRDS based on the methodology of Peters and Taylor (2016). Control variables are defined in the caption of Table 1. $t$ statistics based on heteroskedasticity-consistent and firm-level-clustered standard errors are provided in parentheses. Significance levels: * $p < 0.05$, ** $p < 0.01$. 

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as our previous regressions. We note that reported R&D expenses, while directly measuring growth opportunities, suffer from relatively low coverage in Compustat data. We obtain the same results if we take an alternative approach and substitute missing R&D expenses for zero.

As another robustness check, we regress Jensen and Murphy’s PPS on Capital Expenditures, along with the same set of controls for manager and firm characteristics. Results of these regressions are in Table 6. The dependent variable is again the logarithm of dollar-to-dollar pay sensitivity. The coefficient of 0.391 on Capital Expenditure in column (3) states that a one standard deviation increase in Capital Expenditures is associated with a 2.2% increase in PPS. An important thing to note is that, because capital expenditures represent the exercise of growth options, the expected sign of our estimate is reversed. An increase in growth options leads to a decrease in PPS, and so the exercise of growth options leads to an increase in PPS. We obtain an estimate of similar magnitude when we replace Capital Expenditures with Residual Capital Expenditures as a dependent variable. Residual Capital Expenditures are obtained from fitting an AR(1) model to a firm’s capital expenditures, and capture the unanticipated or discretionary portion of a firm’s investments. In situations where a large portion of a firm’s investments are recurring or reflect ongoing commitments, it is the incidence of new projects that is informative about the exercise of growth options.

The presented regression coefficients are from OLS and fixed-effects models. Similar results obtain in random-effects model.
Table 5: Investment opportunities (R&D) and pay-performance sensitivity

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<th>Dependent variable: PPS</th>
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<td>-0.824**</td>
<td>-0.562*</td>
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<td>-0.308**</td>
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<td>(-3.71)</td>
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<tr>
<td>Tangibility</td>
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<td>Profitability</td>
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<td>-0.128</td>
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<td>(6.30)</td>
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<td>Dividend Paying</td>
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<td>(-5.05)</td>
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<td>Fraction of Inside Directors</td>
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<td>1.741**</td>
<td>0.344**</td>
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<td>(70.34)</td>
<td>(15.28)</td>
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<tr>
<td>Female</td>
<td>-0.261**</td>
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<td>(-7.14)</td>
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<td></td>
</tr>
<tr>
<td>Industry Dummies</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Year Dummies</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Observations          | 67180  | 38125  | 38129  |
$R^2$                  | 0.277  | 0.527  | 0.121  |

The sample covers all executives and firms in Execucomp from 1992 to 2015 and is merged with Compustat data. The dependent variable is the logarithm of the dollar-to-dollar pay-performance sensitivity. Control variables are defined in the caption of Table 1. $t$ statistics based on heteroskedasticity-consistent and firm-level-clustered standard errors are provided in parentheses. Significance levels: * $p < 0.05$, ** $p < 0.01$.  

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Table 6: Investment opportunities (Capital Expenditure) and pay-performance sensitivity

<table>
<thead>
<tr>
<th>Dependent variable: PPS</th>
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<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
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<td>Capital Expenditure</td>
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<td>1.622**</td>
<td>0.391**</td>
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<td></td>
<td>(1.99)</td>
<td>(5.04)</td>
<td>(2.68)</td>
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<td>(-46.15)</td>
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<td>(-4.70)</td>
<td>(-1.65)</td>
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<td>-0.268**</td>
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<td>(-6.44)</td>
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<td>-0.136**</td>
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<td>-0.268**</td>
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<tr>
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<tr>
<td>Year Dummies</td>
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<td>Yes</td>
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</table>

Observations: 151830, 89646, 89646

R²: 0.278, 0.504, 0.117

The sample covers all executives and firms in Execucomp from 1992 to 2015 and is merged with Compustat data. The dependent variable is the logarithm of the dollar-to-dollar pay-performance sensitivity. Control variables are defined in the caption of Table 1. t statistics based on heteroskedasticity-consistent and firm-level-clustered standard errors are provided in parentheses. Significance levels: * p < 0.05, ** p < 0.01.
Table 7: Investment opportunities (Capital Expenditure Residual) and pay-performance sensitivity

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<th>(3)</th>
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<td>(1.90)</td>
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<td>Fraction of Inside Directors</td>
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The sample covers all executives and firms in Execucomp from 1992 to 2015 and is merged with Compustat data. The dependent variable is the logarithm of the dollar-to-dollar pay-performance sensitivity. Capital Expenditure Residual is calculated as the residual from a one-lag firm-specific auto-regressive model of expected scaled capital expenditures. Control variables are defined in the caption of Table 1. $t$ statistics based on heteroskedasticity-consistent and firm-level-clustered standard errors are provided in parentheses. Significance levels: * $p < 0.05$, ** $p < 0.01$. 
5 Conclusion

We analyze a situation in which an investor needs a manager to operate a firm. In our setting, the investor would like the manager to exert costly effort and grow the firm, but is unable to directly observe whether she exerts the recommended effort. To incentivize the recommended effort level, the investor provides the manager with exposure to firm cash flows as part of the manager’s compensation package. The investor has an option to increase the firm’s capital level and this option introduces convexity to the investor’s value function. We find the optimal contract between the investor and the manager and analyze the manager’s incentives in this setting.

An optimal contract provides the manager with sensitivity to the firm’s performance through exposure to unexpected cash flow shocks. Because of the convexity of the firm’s value function, the dynamics of manager sensitivity to cash flow or output differ from those of manager sensitivity to firm value. We develop conditions under which decreasing manager sensitivity to firm value occurs alongside constant or increasing manager sensitivity to cash flows. Despite changing pay performance sensitivity, the manager continues to exert the same recommended effort. The effect is driven by changes in the sensitivity of firm value to manager effort. A natural channel driving variation in this sensitivity is the presence of investment opportunities. We go on to document evidence consistent with our model. Pay performance sensitivity is strongly and negatively related to proxies for growth options.

Our model suggests that the puzzle of low managerial incentives is due in part to value-based measures of pay performance sensitivity being a confounded measure of manager incentives. Decreases in Jensen and Murphy’s PPS can be arise out of optimal contracting, and can be coincident with increases in output-based compensation sensitivity. The observed variation in PPS occurs alongside unobserved variation in the ability of a manager to effect firm value, and incentives depend on both.

While our model provides clean results on managerial incentives, we acknowledge that a variety of other factors may interact with and complicate real world manager compensation.
In particular, the origination of growth options at a firm, which we take as exogenously given, is itself a decision made by firms in an earlier period. The effect of contracting problems on investments in both physical capital and growth options is an interesting and important area of further research.

Growth options are a natural source of variation in sensitivity of firm value to managerial effort between firms and over time. Nevertheless, our key insight is independent of the growth option mechanism, and relies only on variation in the aforementioned sensitivity. This unobserved variation makes PPS an incomplete measure of managerial incentives. Alternative channels that generate this same incompleteness are ready and plentiful. For instance, default-able debt in the capital structure of a firm results in very similar dynamics as in our model, with sensitivity of value to effort increasing as the distance to default decreases. Other channels may have additional implications for managerial incentives and firm value, further complicating our ability to draw conclusions from observed manager pay-performance sensitivity. Given the political and economic resources devoted to determining CEO compensation, there is significant societal interest in ensuring that, in striving to incentivize managers, we are doing so in the proper way.
Appendix A

Proof of Lemma 3 and Verification of Incentive Compatibility

We restrict the manager’s consumption plan to satisfy the following integrability and transversality conditions

\[
E \left[ \int_0^\infty -e^{-r_s}u(\tilde{c}_s, \tilde{a}_s) \, ds \right] < \infty \tag{34}
\]

\[
\lim_{t \to \infty} S_t \overset{a.s.}{=} 0. \tag{35}
\]

Consider an arbitrary contract, comprised of the tuple \((\beta_t, a_t, \tau)\), and note that, if \(W_t\) solves Equation (16), then \(W_t\) is equal, by construction, to the manager’s continuation utility from choosing savings \(S_t = 0\) and effort \(a_t\). Now suppose \(\beta_t\) and \(a_t\) satisfy Equation (18) and consider an arbitrary consumption and effort policy \((\tilde{c}_t, \tilde{a}_t)\). Let

\[
G_t = \int_0^t e^{-r_s}u(\tilde{c}_s, \tilde{a}_s) \, ds + e^{-rt}e^{-\gamma r S_t}W_t, \tag{36}
\]

where \(S_t = \int_0^t e^{r}(t - s)(c_s - \tilde{c}_s) \, ds\) is the manager’s accumulated savings at the point he chooses the alternative consumption plan. An application of Itô’s Lemma gives

\[
e^{rt+\gamma r S_t}dG_t = \left( -\gamma r W_t(c_t - \tilde{c}_t) - \gamma r W_t \beta_t(\tilde{a}_t - a_t) \mu X_t + e^{\gamma r S_t}u(\tilde{c}_t, \tilde{a}_t) \right) dt + \gamma r W_t \beta_t dZ_t. \tag{37}
\]

The \(\tilde{c}_t\) and \(\tilde{a}_t\) that maximize the drift term above must satisfy the following first-order conditions

\[
\gamma r W_t = -e^{\gamma r S_t}u_c(\tilde{c}_t, \tilde{a}_t), \text{ and } \tag{38}
\]

\[
\gamma r W_t \beta_t \mu X_t = -X_t K_t g'(a) e^{\gamma r S_t}u_c(\tilde{c}_t, \tilde{a}_t). \tag{39}
\]
as \( u_a = -u_c X_t K_t g'(a) \). These first-order conditions are solved for \( \tilde{c}_t = c_t + rS_t \) and \( \tilde{a}_t = a_t \) since \( rW_t = u(c_t, a_t) \). Moreover, for \( \tilde{c}_t = c_t + rS_t \) and \( \tilde{a}_t = a_t \), the drift term is zero. Thus, for all other choices of consumption and effort the drift term is weakly negative and \( G_t \) is a super-martingale.

Now consider the manager’s value from choosing the policy \((\tilde{c}_t, \tilde{a}_t)\)

\[
\mathbb{E} \left[ \int_0^\infty e^{-r s} u(\tilde{c}_s, \tilde{a}_s) \, ds \right] = \mathbb{E} [G_t] + \mathbb{E} \left[ \int_t^\infty e^{-r s} u(\tilde{c}_s, \tilde{a}_s) \, ds - e^{-r(t + \gamma S_t)} W_t \right] \tag{40}
\]

\[
\leq G_0 + \mathbb{E} \left[ \int_t^\infty e^{-r s} \left( u(\tilde{c}_s, \tilde{a}_s) - e^{\gamma r S_t} u(c_s, a_s) \right) \, ds \right]. \tag{41}
\]

Now note that \( \lim_{t \to \infty} S_t \overset{a.s.}{=} 0 \), so that \( \lim_{t \to \infty} |\tilde{c}_t - c_t| \overset{a.s.}{=} 0 \), which in turn implies that

\[
\lim_{t \to \infty} \int_t^\infty e^{-r s} \left( u(\tilde{c}_s, \tilde{a}_s) - e^{\gamma r S_t} u(c_s, a_s) \right) \, ds \overset{a.s.}{=} 0. \tag{42}
\]

Finally, by the condition given in Equation (34) and Fubini’s Theorem, we can take the limit as \( t \to \infty \) of both sides of Equation (41) to get

\[
\mathbb{E} \left[ \int_0^\infty e^{-r s} u(\tilde{c}_s, \tilde{a}_s) \, ds \right] \leq G_0 + \lim_{t \to \infty} \mathbb{E} \left[ \int_t^\infty e^{-r s} \left( u(\tilde{c}_s, \tilde{a}_s) - e^{\gamma r S_t} u(c_s, a_s) \right) \, ds \right] \tag{43}
\]

\[
= G_0 = W_0. \tag{44}
\]

Therefore, all other consumption and effort plans \((\tilde{c}_t, \tilde{a}_t)\) yield no more utility than \((c_t, a_t)\) to the manager, and the contract is incentive compatible and no savings.

The conditions given are necessary for a contract to be no savings by Lemma 2. To see that the conditions are also necessary for incentive compatibility, consider any contract \((\beta_t, a_t, \tau)\) such that \( \beta_t \) does not satisfy the condition given Equation 18, then the same argument given above shows that the optimal response to such a contract would be to choose \( \tilde{a}_t \neq a_t \).
Proof of Proposition 1

We verify the optimality of the proposed contract in the following steps. In Step 1, we show that we can replace the investor’s maximization problem (Problem 7) with one in which we maximize a function independent of $Y_t$. We then assume that the optimal investment policy must be a threshold rule that satisfies the boundary conditions given in Equations (27) and (28). In Step 2, we consider a fixed investment threshold and verify the solution to the HJB equations solves Problem 7 for this investment threshold. Finally, we note that we have already verified that the proposed contract is incentive compatible and satisfies the no-savings condition in the proof of Lemma 3.

Before we complete these steps, we make the following technical assumption on $\beta_t$

$$
\mathbb{E} \left[ \int_0^\infty \beta_t^2 X_t^2 dt \right] < \infty
$$

(45)

where the expectation is taken with respect to the measure induced by the incentive compatible dynamics of $X_t$ given $\beta_t$. This restriction does not rule out contracts under which the manager has incentives to exert maximal effort forever. However, such contracts would be infinitely costly to implement, so this assumption can be made without loss of generality.

**Step 1:** Let $v(x, w, k)$ be the value to the investor under a given incentive-compatible, no-savings contract $(c, a, \tau)$ with $X_0 = x$ and $W_0 = w$. Note that Lemmas 2 and 3 imply that the compensation process $c_t$ must be given by Equation (23). The investor’s value is simply the present value of the cash flows of the firm, net of compensation to the manager, and so we have

$$
v(x, w, k) = \mathbb{E} \left[ \int_0^\infty e^{-rt} (X_t K_t - c_t) dt + e^{-r\tau} P \mid X_0 = x, Y_0 = -\frac{1}{\gamma r} \ln (-\gamma rw), K_0 = k \right]
$$

(46)

$$
v(x, w, k) = \mathbb{E} \left[ \int_0^\infty e^{-rt} (X_t K_t (1 - g(a_t))) - rY_t dt + e^{-r\tau} P \right]
$$

(47)

$$
v(x, w, k) = \mathbb{E} \left[ \int_0^\infty e^{-rt} X_t K_t (1 - g(a_t)) dt + e^{-r\tau} P \right]
$$

(48)
\[ + \mathbb{E} \left[ e^{-rt} \left( Y_0 + \int_0^t \frac{1}{2} \gamma r \sigma^2 \beta_s^2 X_s^2 ds + \int_0^t \sigma X_t \beta_t dZ_t^u \right) dt \right] \]

where the last line follows from the dynamics of \( Y_t \) given in Equation (22). Evaluating separately the three terms of the last expectation above, we have

\[
\mathbb{E} \left[ \int_0^\infty e^{-rt} Y_0 dt \right] = Y_0, \tag{49}
\]

\[
\mathbb{E} \left[ \int_0^\infty e^{-rt} \int_0^t \frac{1}{2} \gamma r \sigma^2 \beta_s^2 X_s^2 ds dt \right] = \mathbb{E} \left[ \int_0^\infty \int_s^\infty \frac{1}{2} \gamma r \sigma^2 \beta_t^2 X_t^2 dt ds \right] \tag{50}
\]

\[
= \mathbb{E} \left[ \int_0^\infty e^{-rs} \frac{1}{2} \gamma r \sigma^2 \beta_s^2 X_s^2 ds \right], \tag{51}
\]

\[
\mathbb{E} \left[ \int_0^\infty e^{-rt} \mathbb{E} \left[ \int_0^t \sigma X_t \beta_s dZ_t^u \right] dt \right] = \mathbb{E} \left[ \int_0^\infty e^{-rt} \sigma X_t \beta_s t dZ_t^u dt \right] \tag{52}
\]

\[
= 0. \tag{53}
\]

where we exchange the order of integration by Fubini’s Theorem and the the assumption given in Equation (45). Collecting terms gives

\[
v (x, w, k) = \mathbb{E} \left[ \int_0^\infty e^{-rt} \left( X_t K_t \left( 1 - g \left( a_t \right) \right) - \frac{1}{2} \gamma r \sigma^2 \beta_t^2 X_t^2 \right) dt - e^{-r \tau} \right] - y \tag{54}
\]

where \( y = -\frac{1}{2} \ln (-\gamma rw) \). Thus, the investor’s problem (Problem (7)) is equivalent to the following problem

\[
V (X_0, K_0) = \max_{\beta, a, \tau} \mathbb{E} \left[ e^{-rt} \left( X_t K_t \left( 1 - g \left( a_t \right) \right) - \frac{1}{2} \gamma r \sigma^2 \beta_t^2 \right) dt - e^{-r \tau} \right], \tag{55}
\]

such that

\[
dX_t = a_t \mu X_t dt + \sigma X_t dZ_t, \tag{56}
\]

\[
K_t = k_s + (k_b - k_s) \mathbb{1} (t \geq \tau), \tag{57}
\]

\[
\beta_t = \frac{g' \left( a_t \right)}{\mu} K_t \tag{58}
\]
Step 2: Fix an arbitrary investment rule \( \hat{\tau} \). Let \( \hat{V} \) and \( \hat{\beta}_t \) solve

\[
r\hat{V} = \max_\beta \left\{ \mathcal{L} \left( x, k, \hat{V}; \beta, a \right) \right\},
\]

where

\[
\mathcal{L} \left( x, k, V; \beta, a \right) = xk \left( 1 - g \left( a \right) \right) - \frac{1}{2} \gamma r \beta^2 x^2 + a \mu x \frac{dV}{dx} + \frac{1}{2} \sigma^2 x^2 \frac{d^2V}{dx^2}
\]

such that

\[
\beta = \frac{g' \left( a \right)}{\mu} k,
\]

\[
V \left( X_\tau, k_s \right) \overset{a.s.}{=} V \left( X_\tau, k_b \right) - P,
\]

and let \( \hat{c}_t \) be the compensation given by Equation (23) that makes \( \hat{a}_t \) incentive compatible. In other words, \( \left( \hat{\beta}, \hat{a}_t \right) \) is the optimal contract given investment time \( \hat{\tau} \). Now, consider an arbitrary incentive compatible, no-savings contract \( \left( \tilde{\beta}_t, \tilde{a}_t, \tau \right) \) and let

\[
G_t = \int_0^t e^{-rs} \left( \tilde{X}_s \tilde{K}_s \left( 1 - g \left( \tilde{a}_s \right) \right) - \frac{1}{2} \gamma r \beta_s^2 \tilde{X}_s^2 \right) ds + e^{-rt} \hat{V} \left( \tilde{X}_t, \tilde{K}_t \right) + \mathbb{I} \left( \hat{\tau} \leq t \right) e^{-r\hat{\tau}} P,
\]

where \( G_t \) measures the gains in present value at time \( t = 0 \) derived from using \( \left( \tilde{\beta}_t, \tilde{a}_t, \tau \right) \) up to time \( t \), and \( \tilde{X}_t \) and \( \tilde{K}_t \) are the productivity and capital induced by the contract \( \left( \left\{ \tilde{\beta}_t, \tilde{a}_t \right\}, \hat{\tau} \right) \).

Using Itô’s Lemma gives

\[
e^{rt} dG_t = \left( \mathcal{L} \left( \tilde{X}_t, \tilde{K}_t; \tilde{\beta}_t, \tilde{a}_t \right) - r \hat{V} \right) dt + \sigma \tilde{X}_t \frac{d\hat{V}}{dx} dZ_t
\]

\[
+ \left( \hat{V} \left( X_t, k_b \right) - \hat{V} \left( X_t, k_s \right) - P \right) d\hat{N}_t,
\]

where \( d\hat{N}_t = \mathbb{I} \left( t = \hat{\tau} \right) \) is a counting process that measures the arrival of the investment time \( \hat{\tau} \). Note that the drift term given in (64) is always weakly negative by Equation (59), and that the last term of (64) is always zero. Therefore, \( G_t \) is a super-martingale.

35
Now, consider the value from choosing the contract \((\bar{\beta}_t, \bar{a}_t)\). We have

\[
\mathbb{E} \left[ \int_0^\infty \left( \bar{X}_s \bar{K}_s (1 - g (\bar{a}_s)) - \frac{1}{2} \gamma r \sigma^2 \bar{\beta}_s^2 \bar{X}_s^2 \right) ds - e^{-r \tau} P \right] = \mathbb{E} [G_t] + e^{-r \tau} \mathbb{E} \left[ \int_t^\infty e^{-r(s-t)} \left( \bar{X}_s \bar{K}_s (1 - g (\bar{a}_s)) - \frac{1}{2} \gamma r \sigma^2 \bar{\beta}_s^2 \bar{X}_s^2 \right) ds - \hat{V} (\bar{X}_t, \bar{K}_t) \right] \leq G_0 + e^{-r \tau} \mathbb{E} \left[ \int_t^\infty e^{-r(s-t)} \left( \bar{X}_s \bar{K}_s (1 - g (\bar{a}_s)) - \frac{1}{2} \gamma r \sigma^2 \bar{\beta}_s^2 \bar{X}_s^2 \right) ds - \hat{V} (\bar{X}_t, \bar{K}_t) \right].
\]

Now note that, since \(g (\bar{a}_s) \geq 0\) and \(\bar{\beta}_s^2 \bar{X}_s^2 > 0\), we have

\[
\mathbb{E} \left[ \int_t^\infty e^{-r(s-t)} \left( \bar{X}_s \bar{K}_s (1 - g (\bar{a}_s)) - \frac{1}{2} \gamma r \sigma^2 \bar{\beta}_s^2 \bar{X}_s^2 \right) ds \right] \leq \mathbb{E} \left[ \int_t^\infty e^{-r(s-t)} \bar{X}_s \bar{K}_s ds \right] \leq \mathbb{E} \left[ \int_t^\infty e^{-r(s-t)} \bar{X}_s k_b ds \right] \leq \frac{\bar{X}_t k_b}{r - \mu} \quad (68)
\]

where the last inequality states that the firm value is bounded above by the expected present value of gross (of effort and incentive costs) cash flow \(\bar{X}_t \bar{K}_t\) achieved when \(\bar{a}_t = 1\) and \(K_t = k_b\) for all \(t\). Next note that

\[
\hat{V} (x, k) \geq \frac{x k}{r} > 0 \quad (71)
\]

by Equation (59). Therefore,

\[
\mathbb{E} \left[ \int_0^\infty e^{-r s} \left( \bar{X}_s \bar{K}_s (1 - g (\bar{a}_s)) - \frac{1}{2} \gamma r \sigma^2 \bar{\beta}_s^2 \bar{X}_s^2 \right) ds - e^{-r \tau} P \right] \leq G_0 + e^{-r \tau} \mathbb{E} \left[ \frac{\bar{X}_t k_b}{r - \mu} \right] \leq G_0 + e^{-(r - \mu) t} \frac{X_0 k_b}{r - \mu} \quad (72)
\]

where we bound \(\mathbb{E} [\bar{X}_t]\) above by evaluating the expectation under the assumption of perpetual maximum effort, so that \(\bar{X}_t\) is a geometric Brownian motion. Taking limits of both
sides as \( t \to \infty \) gives

\[
E \left[ \int_0^\infty e^{-rs} \left( \tilde{X}_s \tilde{K}_s (1 - g(\tilde{a}_s)) - \frac{1}{2} \gamma r \sigma^2 \tilde{\beta}_s \tilde{X}_s^2 \right) ds - e^{-r\hat{\tau}} P \right] \leq G_0 = \hat{V}(X_0, K_0),
\]  

(74)

and thus we conclude that any contract \((\hat{\beta}, \hat{a}, \hat{\tau})\) yields a weakly lower value than the contract \((\tilde{\beta}, \tilde{a}, \tilde{\tau})\).

**Proof of Proposition 2**

We first note that the manager’s output-based incentives are measured by \( \beta \):

\[
\beta^*(X) = \frac{g'(a^*(X))}{\mu},
\]  

(75)

and the manager’s dollar value-based PPS is given by:

\[
\phi^*(X) = \frac{\beta^*(X)}{V'(X)} = \frac{g'(a^*(X))k}{\mu V'(X)}.  
\]  

(76)

Under the optimal contract, the optimal effort policy \( a^*(X) \) is given by the first-order condition:

\[
-k_s g'(a^*(X)) - \gamma r \left( \frac{\sigma}{\mu} \right)^2 g'(a^*(X))g''(a^*(X))X + \mu V'(X) = 0.
\]  

(77)

Differentiating the first-order condition with respect to \( k_b \) and rearranging gives the expression

\[
\frac{d a^*}{d k_b} = -\frac{\mu V_{X k_b}(X)}{-k_s g''(a^*) - \gamma r \left( \frac{\sigma}{\mu} k_s \right)^2 X \left( g''(a^*)^2 + g'(a^*)g'''(a^*) \right)}. 
\]  

(78)

In the following analysis, we restrict our attention to parameter values such that the maximum \( a^*(X) \) satisfies the second-order condition. We address each measure of PPS separately below.
Output-based PPS

We first show that output-based PPS is increasing in growth options \( k_b \). Differentiating the expression for output-based incentives, we have

\[
\frac{d\beta^*}{dk_b} = \frac{\sigma}{\mu} X_k g''(a^*) \frac{da^*}{dk_b}, \tag{79}
\]

where, using (78), we can see that

\[
\text{sign} \left( \frac{d\beta^*}{dk_b} \right) = \text{sign} \left( \frac{da^*}{dk_b} \right). \tag{80}
\]

In order to determine this, we note that the denominator of (78) is negative by our assumption that the second-order condition for optimality of \( a^* \) holds.

Furthermore, we argue that \( V_{Xk_b} > 0 \). Beginning with the Hamilton-Jacobi-Bellman equation

\[
rV = Xk_b - g(a^*(X)) X - \frac{1}{2} \gamma r \left( \frac{\sigma}{\mu} g'(a^*(X)) X \right)^2 + a^*(X) \mu X \frac{\partial V}{\partial X} + \frac{1}{2} \sigma^2 X^2 \frac{\partial^2 V}{\partial X^2}, \tag{81}
\]

we differentiate with respect to both size \( X \) and growth option intensity \( k_b \) to get

\[
(r - a^*(X) \mu - a^*_X(X) \mu X) V_{Xk_b} = \left( a^*(X) \mu + \sigma^2 \right) XV_{XXk_b} + \frac{1}{2} \sigma^2 X^2 V_{XXXk_b}, \tag{82}
\]

where the Envelope Theorem tells us that the impact of varying \( k_b \) on optimal effort level \( a^*(X) \) can be ignored when taking the derivative. This result is due to the optimality of \( a^* \) and the first-order condition of the Hamilton-Jacobi-Bellman equation.

We invoke a generalized version of the Feynman-Kac formula, provided as Lemma 4 below, to write the function \( V_{Xk_b} \) as the following expectation:
With this, we have our desired result that \( V_{Xk_b} > 0 \), and therefore growth options increase the power of output-based PPS, \( \frac{d\beta^*}{dk_b} > 0 \).

Value-based PPS

Next, consider the effect \( \phi^* \) of varying \( k_b \):

\[
\frac{d\phi^*}{dk_b} = -\mu^2 \left( \frac{\mu^2 + 2\gamma r \sigma^2 X_k s g''(a^*)}{\gamma r \sigma^2 X_k s g''''(a^*)} \right) \frac{da^*}{dk_b}.
\]

From this, we have

\[
\text{sign} \left( \frac{d\phi^*}{dk_b} \right) = \text{sign} \left( -g''''(a^*) \frac{da^*}{dk_b} \right).
\]

This shows that the sign of the effect of \( k_b \) on PSS is the same as on \( \beta \), and thus positive, if \( g''''(a^*) < 0 \) and the opposite, and thus negative, if \( g''''(a^*) > 0 \), thus completing the proof.

**Lemma 4**

**Lemma 4.** Suppose \( X_t \) evolves according to \( dX_t = \mu(X_t) \, dt + \sigma(X_t) \, dZ_t \). Then, for bounded functions \( f : (0, Y] \to \mathbb{R}, r : (0, Y] \to \mathbb{R}^+, \) and \( \Omega : \mathbb{R} \to \mathbb{R} \), a function \( F : (0, Y] \to \mathbb{R} \) solves both:

\[
r(X) F(X) = f(X) + \mu(X) F_X(X) + \frac{1}{2} \sigma(X)^2 F_{XX}(X),
\]

with a boundary condition \( F(Y) = \Omega(Y) \) and

\[
F(X) = E \left[ \int_0^t e^{-r(X_s)ds} f(X_t) \, dt + e^{-r(X_t)ds} \Omega(Y) \mid X_0 = X \right],
\]

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where \( \tau = \inf \{ t \geq 0 \mid X_t \geq Y \} \).

**Proof of Lemma 4.** The proof essentially follows the proof of Lemma 4 in DeMarzo and Sannikov (2006). Suppose that \( V \) solves equation (86) and define a process \( H_t \) by:

\[
H_t = \int_0^t e^{-\int_s^t r(X_u) \, du} f(X_s) \, ds + e^{-\int_0^t r(X_s) \, ds} V(X_s).
\]

An application of Itô’s formula gives the dynamics for \( H_t \) as:

\[
e^{\int_0^t r(X_s) \, ds} dH_t = \left( f(X_t) + \mu(X_t) V_X(X_t) + \frac{1}{2} \sigma(X_t)^2 V_{XX}(X_t) - r(X_t) V(X_t) \right) dt \\
+ \sigma(X_t) V(X_t) dZ_t.
\]

By Equation (86), the drift of \( H_t \) is zero, and \( H_t \) is a martingale. Since \( V(X) \) is bounded on the interval \([0, \bar{X}]\), \( H_\tau \) is a martingale and \( V \) satisfies:

\[
V(X_0) = H_0 = E[X_\tau \mid X_0] = E \left[ \int_0^\tau e^{-\int_s^\tau r(X_u) \, du} f(X_t) \, dt + e^{-\int_0^\tau r(X_s) \, ds} V(X_\tau) \mid X_0 \right] \\
= E \left[ \int_0^\tau e^{-\int_s^\tau r(X_u) \, du} f(X_t) \, dt + e^{-\int_0^\tau r(X_s) \, ds} \Omega(Y) \mid X_0 \right],
\]

where the last equality follows from the definition of \( \tau \) as a stopping time and the boundary condition \( V(Y) = \Omega(Y) \). \( \square \)
Appendix B. Definition of Variables

*Advertisement.* This variable is advertising expense/total assets = XAD/AT. *Advertisement Missing* is an indicator variable for whether this measure was missing data.

*Capital Expenditures.* This variable is capital expenditures / total assets = CAPX/AT.

*CEO.* This variable is an indicator variable for whether the manager in question is the CEO of the firm.

*CEO Chair.* This variable is an indicator variable for whether the CEO is also chairman of the board.

*Dividend Paying.* This variable is an indicator variable for whether dividends on common stock (DVC) is strictly positive.

*Female.* This variable is an indicator for whether the manager is female. *Firm Age.* This variable equals the year of the data entry less the first year the firm appeared in the CRSP database.

*Firm Size.* This variable is the natural log of total assets = log(AT).

*Fraction of Inside Directors.* This variable is the number of inside board directors divided by board size. Inside directors are those who personally or had a family member serve as a current or former firm manager.

*Leverage.* This variable is (long term debt + short term debt)/total assets = (DLTT + DLC)/AT.

*Market-to-Book.* This variable equals (market value of equity plus book value of debt)/book value of assets = (CSHO \( \times \) PRCC_F + AT - CEQ)/AT.

*Profitability.* This variable is operating income before D&A/total assets = OIVDP/AT.
R&D. This variable equals R&D expense/book value of assets = XRD/AT.

Tangibility. This variable equals net PP&E/total assets = PPENT/AT.

Tobin’s Q. This variable is the Peters Taylor measure of Total Q found on WRDS = Q_TOT.

Value-to-Book. First, we regress log(market value of equity plus book value of debt) = log(CSHO × PRCC_F + AT - CEQ) on log(book value) of assets (log(AT)), including an industry fixed effect, where industry is determined by four-digit SIC codes. Second, we subtract log book value of assets (log(AT)) from the fitted values from the regression.
References


