Abstract. We study the nature of deflation risk by extracting the objective distribution of inflation from the market prices of inflation swaps and options. We find that the market expects inflation to average about three percent over the next 30 years. Despite this, the market places substantial probability weight on deflation scenarios in which prices decline by more than 10 to 20 percent over extended horizons. We find that the market prices the economic tail risk of deflation similarly to other types of tail risks such as corporate default or catastrophic insurance losses. In contrast, inflation tail risk generally has a smaller premium. Short-term deflation risk is significantly linked to measures of systemic financial risk while longer-term deflation risk is driven more by sovereign credit and macroeconomic factors.
1. INTRODUCTION

Deflation has played a central role in the worst economic meltdowns experienced in US history. Key examples include the deflations associated with the Panic of 1837, the Long Depression of 1873–1896, and the Great Depression of the 1930s. In light of this, it is not surprising that deflation is now one of the most-feared risks facing participants in the financial markets. In recent years, the financial press has increasingly raised concerns about a global deflationary spiral and has used terms such as “nightmare scenario” or “looming disaster” to describe the growing threat.\(^1\) Furthermore, addressing the risk of deflation was one of the primary motivations behind a number of actions taken by the Federal Reserve in recent years such as unconventional monetary policy and quantitative easing programs.\(^2\)

Despite the severe potential effects of deflation, relatively little is known about how large the risk of deflation actually is, or about the economic and financial factors that contribute to deflation risk. The primary reason for this may simply be that deflation risk has traditionally been very difficult to measure. For example, as shown by Ang, Bekaert, and Wei (2007) and others, econometric models based on the time series of historical inflation perform poorly even in estimating the first moment of inflation. In addition, while surveys of inflation tend to do better, these surveys are limited to forecasts of expected inflation over shorter horizons and provide little or no information about the tail probability of deflation.

This paper presents a simple market-based approach for measuring deflation risk. This approach allows us to solve directly for the market’s assessment of the probability of deflation for horizons of up to 30 years using the prices of inflation swaps and options. In doing this, we first use standard techniques to infer the risk-neutral density of inflation that underlies the prices of inflation calls and puts. We then use maximum likelihood to estimate the inflation risk premium embedded in the term structure of inflation swap rates using methods familiar


\(^{2}\) For example, see Bernanke (2012), “Monetary Policy since the Onset of the Crisis,” www.federalreserve.gov/newsevents/speech/bernanke20120831a.htm.
from the affine term-structure literature. Finally, we solve for the actual or objective distribution of inflation by adjusting for the estimated risk premium. A key advantage of this approach is that we recover the entire distribution of inflation rather than just the first moment or expected inflation. This is important since this allows us to measure the probability of tail events such as deflation.

We find that the inflation risk premium varies significantly over time and can be both positive and negative in value. In fact, it may often be more accurate to characterize the risk premium as a deflation risk premium. This is particularly true during the European sovereign crisis when the premium becomes sharply negative. On average, the risk premium is close to zero for horizons ranging from one to 20 years. We also find that the market expects inflation of roughly 2.50 to 3.00 percent for horizons ranging from five to 30 years. Note that the horizons of up to 30 years for these market-implied forecasts far exceed those of existing survey-based forecasts.

We solve for the probability of deflation over horizons ranging up to 30 years directly from the distribution of inflation. The empirical results are very striking. We find that the market places a significant amount of weight on the probability that deflation occurs over extended horizons. Furthermore, the market-implied probability of deflation can be substantially higher than that estimated by policy makers. For example, in a speech on August 27, 2010, Federal Reserve Chairman Ben S. Bernanke stated that “Falling into deflation is not a significant risk for the United States at this time.” On the same date, the market-implied probability of deflation was 27.35 percent for a two-year horizon, 16.15 percent for a five-year horizon, and 6.39 percent for a ten-year horizon. These probabilities are clearly not negligible. On average, the market-implied probability of deflation during the sample period was 11.36 percent for a two-year horizon, 6.34 percent for a five-year horizon, 5.23 percent for a ten-year horizon, and 2.77 percent for a 30-year horizon. The risk of deflation, however, varies significantly and these probabilities have at times been substantially larger than the averages. In particular, large increases in the probability of deflation often coincide with major events in the financial markets such as the ratings downgrades of Spain in 2010 or the downgrade of US Treasury debt by Standard and Poors in August 2011.

Deflation is clearly an economic tail risk and changes in deflation risk may reflect the market’s fears of a meltdown scenario. Thus, a natural next step is to

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4Note that we are interpreting tail risk as including more than just event risk or jump risk. Event or jump risks are adverse economic events that occur relatively
examine whether deflation risk is related to other serious types of tail risk in the financial markets or in the macroeconomy in general. Focusing first on the pricing of deflation risk, we find that the ratio of the risk-neutral probability of deflation to the actual probability of deflation is often on the order of 1.30. Similarly, the ratios for scenarios of deflation of one and two percent per year range from about 1.50 to 2.00. These ratios of risk-neutral to objective probabilities are similar to those for other types of tail risk. For example, Froot (2001) finds that the ratio of the price of catastrophic reinsurance to expected losses is on the order of two or higher. Driessen (2005), Berndt, Duffie, Douglas, Ferguson, and Schranz (2005), Giesecke, Longstaff, Schaefer, and Strebulaev (2011) estimate that the ratio of the price of expected losses on corporate bonds to actual expected losses is also about two. These findings are also consistent with models with rare consumption disasters, such as that pioneered by Rietz (1988) and further developed by Longstaff and Piazzesi (2004), Barro (2006), and Gourio (2008), which were explicitly engineered to produce high risk-neutral probabilities for rare consumption disasters such as the Great Depression. Recently, Barro has argued that this class of models can account for the equity premium when calibrated to the 20th Century experience of developed economies. Gabaix (2012) and Wachter (2013) have extended these models to incorporate a time-varying intensity of consumption disasters. This extension delivers bond and stock market return predictability similar to what is observed in the data.

We next consider the relation between deflation risk and specific types of financial and macroeconomic tail risks that have been described in the literature. In particular, we consider a number of measures of systemic financial risk, collateral revaluation risk, sovereign default risk, and business cycle risk and investigate whether these are linked to deflation risk. We find that the nature of deflation risk varies with horizon. Short-term deflation risk is strongly related to measures of risk in the financial markets such as Libor spreads, swap spreads, stock returns, and stock market volatility. Intermediate-term deflation risk is more related to structural factors such as the risk of sovereign defaults in the Eurozone. In contrast, long-term deflation risk is driven primarily by macroeconomic factors. These results are consistent with both models of systemic financial risk as well a number of classical macroeconomic theories about the relation between prices and employment. Overall, these results support the view that the risk of macroeconomic shocks which result in deflation is closely related to tail risks in financial markets. Thus, option prices are highly informative about the probability the market imputes to these rare disaster states, arguably more informative than quantity data (see, for example, recent work by Backus, Chernov, and Martin (2011) using equity options). Our inflation option suddenly. In contrast, tail risk can also include extreme scenarios with severe economic consequences which may unfold over extended periods.
findings imply that market participants mostly expect deflation in the US in these disaster states. This is consistent with US historical experience in which depressions/deflationary spirals have been associated with major collapses in the financial system.

Finally, we also compute the probabilities of inflation exceeding various thresholds. The results indicate that while the probability of inflation in the near term is relatively modest, the long-term probabilities of inflation are much higher. Interestingly, we find inflation tail risk is priced much more modestly than is deflation tail risk, with ratios of risk-neutral to actual probabilities on the order of 1.10 to 1.30.

From a broader perspective, the approach and results of this paper have a number of other important economic implications. First, these results illustrate that the information contained in the market prices of derivatives such as inflation swaps and options provides a powerful lens for studying macroeconomic variables. In a literal sense, these types of markets may make it possible to study the effects of changes in fiscal and monetary policy in real time, potentially opening a window on high-frequency macroeconomic analysis conducted at intra-day time scales.

Second, finding that the inflation risk premium sometimes switches signs and becomes a deflation risk premium raises an intriguing question: what would be the effect of the monetary authority targeting the inflation risk premium? In particular, if monetary policy were conducted in a way that resulted in the inflation/deflation risk premium being zero, would this result in a better balance between the Federal Reserve’s dual mandate of maximum employment and stable prices? Intuitively, the zero-risk-premium (ZRP) point could signal that the market’s perception of the risk of deflation (with its resulting macroeconomic ills) was just offset by the risk of inflation. Our results suggest that ZRP-targeting may be worth considering as alternative way of conducting monetary policy.

Third, our results also have implications for Treasury debt management. In particular, whenever the Treasury issues Treasury Inflation Protected Securities (TIPS), the Treasury essentially writes an at-the-money deflation put and packages it together with a standard inflation-linked bond. The returns on writing these deflation puts are potentially large because of the substantial risk premium associated with deflation tail risk. If the Treasury is better suited to bear deflation tail risk than the marginal investor in the market for inflation protection, then providing a deflation put provides an extra source of revenue for the Treasury that is non-distortionary. There are good reasons to think that the Treasury is better equipped to bear deflation risk, not in the least because the Treasury
and the Federal Reserve jointly control the price level.\footnote{Since the ratio of risk-neutral to actual probabilities is larger for deflation than for high-inflation scenarios, this same logic is not as applicable to the standard inflation protection built into TIPS.}


Two important recent papers have parallels to our work. Christensen, Lopez, and Rudebusch (2011) fit an affine term-structure model to the Treasury real and nominal term structures and estimate the value of the implicit deflation option embedded in TIPS prices. Our research significantly extends their results by estimating deflation probabilities for horizons out to 30 years directly using market inflation option prices. Kitsul and Wright (2013) also use inflation options to infer the risk-neutral density for inflation, but do not formally solve for the objective density of inflation. Our results corroborate and significantly extend their results.

The remainder of this paper is organized as follows. Section 2 briefly discusses the history of deflation in the United States. Section 3 provides an introduction to the inflation swap and options markets. Section 4 presents the inflation model used to value inflation derivatives. Section 5 discusses the maximum likelihood estimation of the inflation model. Section 6 describes the distribution of inflation. Section 7 considers the implications of the results for deflation probabilities and the pricing of deflation risk. Section 8 examines the relation between deflation risk and other types of financial and macroeconomic tail risks. Section 9 presents results for the probabilities of several inflation scenarios. Section 10 summarizes the results and makes concluding remarks.

**2. DEFLATION IN US HISTORY**
The literature on deflation in the US is far too extensive for us to be able to review in this paper. Key references on the history of deflation in the US include North (1961), Friedman and Schwartz (1963), and Atack and Passell (1994). We will simply observe that deflation was a relatively frequent event during the 19th Century, but has diminished in frequency since then. Bordo and Filardo (2005) report that the frequency of an annual deflation rate was 42.4 percent from 1801–1879, 23.5 percent from 1880–1913, 30.6 percent from 1914–1949, 5.0 percent from 1950–1969, and zero percent from 1970–2002. The financial crisis of 2008–2009 was accompanied by the first deflationary episode in the US since 1955.

Economic historians have identified a number of major deflationary episodes. Key examples include the crisis of 1815–1821 in which agricultural prices fell by nearly 50 percent. The banking-related Panic of 1837 was followed by six years of deflation in which prices fell by nearly 30 percent. The post-Civil-War greenback period experienced a number of severe deflations and the 1873–1896 period has been called the Long Depression. This period experienced massive amounts of corporate bond defaults and Friedman and Schwartz (1963) estimate that the price level declined by 1.7 percent per year from 1875 to 1896. The US suffered a severe deflationary spiral during the early stages of the Great Depression in 1929–1933 as prices rapidly fell by more than 40 percent.

Although Atkeson and Kehoe (2004), Bordo and Filardo (2005), and others show that not all deflations have been associated with severe declines in economic output, a common thread throughout US history has been that deflationary episodes are typically associated with turbulence or crisis in the financial system.

3. THE INFLATION SWAPS AND OPTIONS MARKETS

In this section, we begin by reviewing the inflation swaps market. We then provide a brief introduction to the relatively new inflation options market.

3.1 Inflation Swaps

As discussed by Fleckenstein, Longstaff, and Lustig (2014), US inflation swaps were first introduced when the Treasury began auctioning TIPS in 1997, and have become increasingly popular among institutional investment managers. Pond and Mirani (2011) estimate the notional size of the inflation swap market to be on the order of hundreds of billions.

In this paper, we focus on the most widely-used type of inflation swap which is designated a zero-coupon swap. This swap is executed between two counterparties at time zero and has only one cash flow which occurs at the maturity date.
of the swap. For example, imagine that at time zero, the ten-year zero-coupon inflation swap rate is 300 basis points. As is standard with swaps, there are no cash flows at time zero when the swap is executed. At the maturity date of the swap in ten years, the counterparties to the inflation swap exchange a cash flow of \((1 + 0.0300)^{10} - I_T\), where \(I_T\) is the relative change in the price level between now and the maturity date of the swap. The timing and index lag construction of the inflation index used in an inflation swap are chosen to match precisely the definitions applied to TIPS issues.

The zero-coupon inflation swap rate data used in this study are collected from the Bloomberg system. Inflation swap data for maturities ranging from one to 30 years are available for the period from July 23, 2004 to July 29, 2014. Data for inflation swaps with maturities of up to 55 years are available beginning later in the sample. Recent research by Fleming and Sporn (2012) concludes that “the inflation swap market appears reasonably liquid and transparent despite the market’s over-the-counter nature and modest activity.” They estimate that realized bid-ask spreads for customers in the inflation swap market are on the order of three basis points. Conversations with inflation swap traders confirm that these instruments are fairly liquid with typical bid-ask spreads consistent with those reported by Fleming and Sporn. Table 1 presents summary statistics for the inflation swap rates.

As shown, average inflation swap rates range from 1.74 percent for one-year inflation swaps, to a high of 2.89 percent for 30-year inflation swaps. The volatility of inflation swap rates is generally declining in the maturity of the contracts. The dampened volatility of long-horizon inflation swap rates suggests that the market may view expected inflation as being mean-reverting in nature. Table 1 also shows that there is evidence of deflationary concerns during the sample period. For example, the one-year swap rate reached a minimum of −4.545 percent during the height of the 2008 financial crisis amid serious fears about the US economy sliding into a full-fledged depression/deflation scenario.

### 3.2 Inflation Options

The inflation options market had its inception in 2002 with the introduction of caps and floors on the realized inflation rate. While trading in inflation options was initially muted, the market gained considerable momentum as the financial crisis emerged and total interbank trading volume reached $100 billion. While the inflation options market is not yet as liquid as, say, the stock index options market, the market is sufficiently liquid that active quotations for inflation cap and floor prices have been readily available in the market since 2009 for a wide

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\footnote{For a discussion of the inflation derivatives markets, see Jarrow and Yildirim (2003), Mercurio (2005), Kerkoff (2005), and Barclay’s Capital (2010).}
In Europe and the United Kingdom, insurance companies are among the most active participants in the inflation derivatives market. In particular, much of the demand in ten- and 30-year zero percent floors is due to pension funds trying to protect long inflation swaps positions. In contrast, insurance companies and financial institutions that need to hedge inflation risk are the most active participants on the demand side in the US inflation options market.

The most-actively traded inflation options are year-on-year and zero-coupon inflation options. Year-on-year inflation options are caps and floors that pay the difference between a strike rate and annual inflation on an annual basis. Zero-coupon options, in contrast, pay only one cash flow at the expiration date of the contract based on the cumulative inflation from inception to the expiration date. To illustrate, assume that the realized inflation rate over the next ten years was two percent. A ten-year zero-coupon cap struck at one percent would pay a cash flow of $\max(0, 1.0200^{10} - 1.0100^{10})$ at its expiration date. In this paper, we focus on zero-coupon inflation options since their cash flows parallel those of zero-coupon inflation swaps.

As with inflation swaps, we collect inflation cap and floor data from the Bloomberg system. Data are available for the period from October 5, 2009 to July 29, 2014 for strikes ranging from negative two percent to six percent in increments of 50 basis points. We check the quality of the data by insuring that the cap and floor prices included satisfy standard option pricing bounds such as those described in Merton (1973) including put-call parity, monotonicity, intrinsic value lower bounds, strike price monotonicity, slope, and convexity relations. To provide some perspective on the data, Table 2 provides summary statistics for call and put prices for selected strikes.

As illustrated, inflation cap and floor prices are quoted in basis points, or equivalently, as cents per $100 notional. Interestingly, inflation option prices are not always monotonically increasing in maturity. This may seem counterintuitive given standard option pricing theory, but is it important to recognize that the inflation rate is a macro variable rather than the price of a traded asset. For most maturities, we have about 20 to 25 separate cap and floor prices with strikes varying from negative two percent to six percent from which to estimate the risk-neutral density of inflation.

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We observe that similar nonmonotonic behavior occurs with interest rate options such as interest rate caps, floors, and swaptions; see Longstaff, Santa-Clara, and Schwartz (2001).
4. MODELING INFLATION

In this section, we present the continuous time model used to describe the dynamics of inflation under both the objective and risk-neutral measures. We also describe the application of the model to the valuation of inflation swaps and options.

4.1 The Inflation Model

We begin with a few key items of notation. For notational simplicity, we will assume that all inflation contracts are valued as of time zero and that the initial price level at time zero is normalized to one. Furthermore, time zero values of state variables are unsubscripted. Let $I_t$ denote the relative change in the price level from time zero to time $t$.

Under the objective or actual measure $P$, the dynamics of the price level are given by

\[
\begin{align*}
\text{d}I &= X \text{ } I \text{ } \text{d}t + \sigma \text{ } I \text{ } \text{d}Z_I, \\
\text{d}X &= \kappa \text{ } (Y - X) \text{ } \text{d}t + \eta \text{ } \text{d}Z_X, \\
\text{d}Y &= (\mu - \beta Y) \text{ } \text{d}t + s \text{ } \text{d}Z_Y.
\end{align*}
\]

In this specification, $X_t$ represents the instantaneous expected inflation rate. The state variable $Y_t$ represents the long-run trend in expected inflation towards which the process $X_t$ reverts. The processes $Z_I$, $Z_X$, and $Z_Y$ are standard Brownian motions and are assumed to be uncorrelated with each other. This affine specification has parallels to the long-run risk model of Bansal and Yaron (2004) and allows for a wide range of possible time series properties for realized inflation.

Under the risk-neutral valuation measure $Q$, the dynamics of the price level are given by

\[
\begin{align*}
\text{d}I &= X \text{ } I \text{ } \text{d}t + \sigma \text{ } I \text{ } \text{d}Z_I, \\
\text{d}X &= \lambda \text{ } (Y - X) \text{ } \text{d}t + \eta \text{ } \text{d}Z_X, \\
\text{d}Y &= (\alpha - \beta Y) \text{ } \text{d}t + s \text{ } \text{d}Z_Y.
\end{align*}
\]

\(^8\text{Since the initial price level equals one, we will further simplify notation by not showing the dependence of valuation expressions on the initial price level } I.\)
where the parameters $\lambda$ and $\alpha$ that now appear in the system of equations allow for the possibility that the market incorporates time-varying inflation-related risk premia into asset prices. In particular, the model allows the risk-neutral distributions of $I$, $X$, and $Y$ to differ from the corresponding distributions under the actual dynamics. Thus, the model permits a fairly general structure for inflation risk premia.

It is important to acknowledge, however, that this model of inflation dynamics is a relatively simple one. In particular, the model implies that the log inflation rate is normally distributed under both the actual and risk-neutral measures (albeit with different means and variances under each measure). Thus, more complex features such as jumps or stochastic volatility in the inflation process are absent from this specification. In an earlier version of the paper, however, we used a much more general specification for the inflation process that was consistent with jumps and stochastic volatility and allowed the distribution of inflation to be in the class of five-parameter generalized hyperbolic distributions. The results obtained from the more general specification were very similar to those we obtain using this simpler and more intuitive specification. Thus, the results appear to be fairly robust to the choice of the dynamic inflation model.

Finally, let $r_t$ denote the nominal instantaneous riskless interest rate. We can express this rate as $r_t = R_t + X_t$ where $R_t$ is the real riskless interest rate and $X_t$ is expected inflation. For tractability, we also assume that $R_t$ is uncorrelated with the other state variables $I_t$, $X_t$, and $Y_t$.

4.2 Valuing Inflation Swaps

From the earlier discussion, an inflation swap pays a single cash flow of $I_T - F$ at maturity date $T$, where $F$ is the inflation swap price set at initiation of the contract at time zero. Note that $F = (1 + f)^T$ where $f$ is the inflation swap rate. The Appendix shows that the inflation swap price can be expressed in closed form as

$$F(X, Y, T) = \exp (-A(T) - V(T) - B(T)X - C(T)Y), \quad (7)$$

where
\[ A(T) = \frac{\alpha \lambda}{\beta - \lambda} \left( \frac{1}{\beta} (T - \frac{1}{\beta} (1 - e^{-\beta T})) - \frac{1}{\lambda} (T - \frac{1}{\lambda} (1 - e^{-\lambda T})) \right) , \] \( (8) \)

\[ B(T) = -\frac{(1 - e^{-\lambda T})}{\lambda} , \] \( (9) \)

\[ C(T) = \frac{\lambda}{\beta - \lambda} \left( \frac{1}{\beta} (1 - e^{-\beta T}) - \frac{1}{\lambda} (1 - e^{-\lambda T}) \right) , \] \( (10) \)

\[ V(T) = \frac{s^2 \lambda^2}{2(\lambda - \beta)^2} \left( \frac{1}{\beta^2} (T - \frac{2}{\beta} (1 - e^{-\beta T}) + \frac{1}{2\beta} (1 - e^{-2\beta T})) \right) \]

\[ - \frac{2}{\beta \lambda} (T - \frac{1}{\beta} (1 - e^{-\beta T})) - \frac{1}{\lambda} (1 - e^{-\lambda T}) + \frac{1}{\beta + \lambda} (1 - e^{-(\beta + \lambda)T}) \]

\[ + \frac{1}{\lambda^2} (T - \frac{2}{\lambda} (1 - e^{-\lambda T}) + \frac{1}{2\lambda} (1 - e^{-2\lambda T})) \]

\[ + \frac{\eta^2}{2\lambda^2} \left( T - \frac{2}{\lambda} (1 - e^{-\lambda T}) + \frac{1}{2\lambda} (1 - e^{-2\lambda T}) \right) . \] \( (11) \)

### 4.3 Valuing Inflation Options

Let \( C(X, Y, T) \) denote the time-zero value of a European inflation cap or call option with strike \( K \) and expiration date \( T \). The payoff on this option at its expiration date is \( \max(0, I_T - (1 + K)^T) \). The Appendix shows that the value of the call option at time zero is given by

\[ C(X, Y, T) = D(T) \ F(X, Y, T) \ N(\phi) \]

\[- D(T) \ (1 + K)^T \ N(\phi - \sqrt{2V(T) + \sigma^2 T}) , \] \( (12) \)

where

\[ \phi = \frac{\ln F(X, Y, T) - T \ln(1 + K) + V(T) + \sigma^2 T/2}{\sqrt{2V(T) + \sigma^2 T}} , \] \( (13) \)

and where \( D(T) \) is the price of a riskless discount bond with maturity \( T \). A similar representation holds for the value of an inflation floor or put option \( P(X, Y, T) \) with payoff at expiration date \( T \) of \( \max(0, (1 + K)^T - I_T) \).

### 5. MODEL ESTIMATION
In identifying the distribution of inflation, we follow a simple three-step approach using techniques familiar from the empirical options and affine term-structure literatures. First, we solve for the risk-neutral distribution of inflation embedded in the prices of inflation caps and floors having the same maturity but differing in their strike prices. Second, we identify the inflation risk premia by maximum likelihood estimation of the affine model of the term structure of inflation swaps. Third, we make the transformation from the implied risk-neutral distribution to the objective distribution of inflation.

5.1 Solving for the Risk-Neutral Distribution

There is an extensive literature on the estimation of risk-neutral distributions from option prices. Key examples include Banz and Miller (1978), Breeden and Litzenberger (1978), Longstaff (1995), Aït-Sahalia and Lo (1998), and others. One stream of this literature suggests the use of nonparametric representations of the risk-neutral density. The majority of the literature, however, is based on using general parametric specifications of the risk-neutral density. Since the model specification for inflation dynamics given in Equations (4) through (6) leads to simple closed-form solutions for inflation calls and puts, we can directly solve for the risk-neutral density of inflation from the market prices of inflation options.

We solve for the implied risk-neutral density in the following way. For each date and horizon, we collect prices for all available inflation caps and floors. On average, we have prices for between 20 and 25 caps and floors with strike prices ranging from negative two percent to six percent in steps of 50 basis points. We then solve for the lognormal density that results in the best root mean squared fit to the set of cap and floor prices, while requiring that the model exactly match the corresponding inflation swap rate. We then repeat this process for each day in the sample period and for each horizon of option expirations, one, two, three, five, seven, ten, 20, and 30 years. Although not shown, the algorithm is able to fit the inflation cap and floor prices very accurately. In particular, the model prices are typically within several percent of the corresponding market prices and would likely be well within the actual bid-ask spreads for these options.

5.2 Maximum Likelihood Estimation

As shown in Equation (7), the closed-form solution for inflation swap prices depends only on the two state variables $X$ and $Y$ that drive expected inflation.

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9We fit the model based on the root mean squared pricing errors for the data, but we obtain essentially identical results when the errors are expressed as percentages of the option prices, or when the pricing errors are weighted by the intrinsic values of the options.
An important advantage of this feature is that it allows us to use standard affine term-structure modeling techniques to estimate $X$ and $Y$ and their parameters under both the objective and risk-neutral measures. In doing this, we apply the maximum likelihood approach of Duffie and Singleton (1997) to the term structure of inflation swaps for maturities ranging from one to 30 years (but not the longer maturities for which we do not have complete time series).

Specifically, we assume that the two- and 30-year inflation swap rates are measured without error. Thus, given a parameter vector $\Theta$, substituting these maturities into the log of the inflation swap expression in Equation (7) results in a system of two linear equations

\[
\ln F(X, Y, 2) = -A(2) - V(2) - B(2)X - C(2)Y, \quad (14)
\]
\[
\ln F(X, Y, 30) = -A(30) - V(30) - B(30)X - C(30)Y, \quad (15)
\]

in the two state variables $X$ and $Y$. This means that $X$ and $Y$ can be expressed as explicit linear functions of the two inflation swap prices $F(X, Y, 2)$ and $F(X, Y, 30)$. Let $J$ denote the Jacobian of the mapping from the two swap rates into $X$ and $Y$.

At time $t$, we can now solve for the inflation swap rate implied by the model for any maturity from the values of $X_t$ and $Y_t$ and the parameter vector $\Theta$. Let $\epsilon_t$ denote the vector of differences between the market value and the model value of the inflation swaps for the other maturities implied by $X_t$, $Y_t$, and the parameter vector $\Theta$. Under the assumption that $\epsilon_t$ is conditionally multivariate normally distributed with mean vector zero and a diagonal covariance matrix $\Sigma$ with main diagonal values $\nu_j$ (where the subscripts denote the maturities of the corresponding inflation swaps), the log of the joint likelihood function $LLK_t$ of the two- and 30-year inflation swap prices and $\epsilon_{t+\Delta t}$ conditional on the inflation swap term structure at time $t$ is given by

\[
\begin{align*}
&= -(13/2) \ln(2\pi) + \ln |J_{t+\Delta t}| - \frac{1}{2} \ln |\Sigma| - \frac{1}{2} \epsilon_{t+\Delta t}' \Sigma^{-1} \epsilon_{t+\Delta t} \\
&- \ln(2\pi \sigma_X \sigma_Y \sqrt{1 - \rho_{XY}^2}) - \frac{1}{2(1 - \rho_{XY}^2)} \left[ \frac{X_{t+\Delta t} - \mu_{X_t}}{\sigma_X} \right]^2 \\
&- 2\rho_{XY} \left( \frac{X_{t+\Delta t} - \mu_{X_t}}{\sigma_X} \right) \left( \frac{Y_{t+\Delta t} - \mu_{Y_t}}{\sigma_Y} \right) + \left( \frac{Y_{t+\Delta t} - \mu_{Y_t}}{\sigma_Y} \right)^2,
\end{align*}
\]

(16)

where the conditional moments $\mu_{X_t}$, $\mu_{Y_t}$, $\sigma_X$, $\sigma_Y$, and $\rho_{XY} = \sigma_{XY}/\sqrt{\sigma_X^2 \sigma_Y^2}$ of
$X_{t+\Delta t}$ and $Y_{t+\Delta t}$ are given in the Appendix. The total log likelihood function is given by summing $LLK_t$ over all values of $t$.

We maximize the log likelihood function over the 20-dimensional parameter vector $\Theta = \{\kappa, \eta, \mu, s, \lambda, \alpha, \beta, v_1, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{12}, v_{15}, v_{20}, v_{25}\}$ using a standard quasi-Newton algorithm with a finite difference gradient. As a robustness check that the algorithm achieves the global maximum, we repeat the estimation using a variety of different starting values for the parameter vector. Table 3 reports the maximum likelihood estimates of the parameters and their asymptotic standard errors. The fitting errors from the estimation are all relatively small with the typical standard deviation ranging from roughly six to ten basis points, depending on maturity.

5.3 Solving for the Objective Distribution

Once the maximum likelihood estimates of the both the risk neutral and objective parameters are obtained, we can directly solve for the distribution of inflation under the objective measure. Specifically, the density of $\ln I_T$ under the risk-neutral measure is normal and is thus fully characterized by two moments. Given the maximum likelihood estimates of the parameters, it is straightforward to make the adjustments to the moments of the risk-neutral density to obtain those for the objective density. Since the objective density for $\ln I_T$ is also normal, these two adjusted moments completely characterize the objective density.

6. THE DISTRIBUTION OF INFLATION

As a preliminary to the analysis of deflation risk, it is useful to first present the empirical results for inflation risk premia, expected inflation, and inflation volatility.

6.1 Inflation Densities

To provide some perspective on the nature of the inflation density under the objective measure, Figure 1 plots the time series of inflation densities for several horizons. As shown, there is considerable variation in the shape of the inflation distribution for the shorter horizons. In contrast, the distribution of inflation for longer horizons is more stable over time.

6.2 Inflation Risk Premia

We measure the inflation risk premium by simply taking the difference between the fitted inflation swap and expected inflation rates. This is the essentially the way in which many market participants think about inflation risk premia. When the inflation swap rate is higher than expected inflation, the inflation risk
premium is positive, and vice versa. There is no compelling theoretical reason why the inflation risk premium could not be negative in sign. In this case, the risk premium might well be viewed as a deflation risk premium.

Table 4 presents summary statistics for the average inflation risk premia for horizons ranging from one year to 30 years. Figure 2 plots the time series of inflation risk premia for several horizons. As shown, the magnitudes of the average risk premia are relatively small. For example, the average annualized risk premia for inflation swap contracts with maturities from one to 20 years are less than ten basis points in absolute value. The averages for shorter horizons are slightly negative, while the averages for longer horizons up to 15 years are slightly positive. The average risk premia for the 20-, 25-, 30-year maturities are negative. Table 4 also shows that the inflation risk premia vary through time, and the volatility of inflation risk premia for shorter horizons is higher than for longer horizons.

These inflation risk premia estimates are broadly consistent with previous estimates obtained using alternative approaches and different data sets by other researchers. For example, Haubrich, Pennachi, and Ritchken (2012) estimate the ten- and 30-year inflation risk premia to be 51 and 101 basis points, respectively. Buraschi and Jiltsov (2005) and Campbell and Viceira (2001) estimate the ten-year inflation risk premium to be 70 and 110 basis points, respectively. Ang, Bekaert, and Wei (2008) estimate the five-year inflation risk premium to be 114 basis points. In addition, the fact that all of the estimated risk premia take negative values at some point during the sample period is consistent with the findings of Campbell, Shiller, and Viceira (2009), Bekaert and Wang (2010), Campbell, Sunderam, and Viceira (2013), and others.

Finding that inflation risk premia change signs through time is an intriguing result. Intuitively, one way to think about why the risk premium could change signs is in terms of the link between inflation and the macroeconomy. For example, when inflation risk is perceived to be counter-cyclical, the market price of inflation risk should be positive. This is the regime in which aggregate supply shocks (e.g. oil shocks) account for most of the variation in output: high inflation coincides with low output growth. Thus, investors pay an insurance premium when buying inflation protection in the market.

But this line of reasoning suggests that the sign of the inflation risk premium could be negative when inflation risk was perceived to have become pro-cyclical. In that regime, inflation innovations tend to be positive when the average investor’s marginal utility is high. Hence, a nominal bond provides insurance against bad states of the world, whereas a real bond does not. In this case, investors receive an insurance premium when buying inflation hedges. When aggregate demand shocks account for most of the variation in output growth, then
we would expect to see pro-cyclical inflation: high inflation coincides with high output growth.

The variation in the sign of the inflation risk premium also raises a number of interesting questions about optimal monetary policy. Since the Treasury seems constrained to issue mostly nominal bonds, it strictly prefers lower inflation risk premia. Higher inflation risk premia increase the costs of government debt financing, funded by distortionary taxes. In particular, when inflation risk premia are negative, issuing nominal bonds is very appealing. However, since the level of expected inflation is related to the size of the inflation risk premium, the government may have the ability to influence the risk premium to some degree by how it targets inflation.

6.3 Expected Inflation

To solve for the expected inflation rate for each horizon, we use the inflation swap rates observed in the market and adjust them by the inflation risk premium implied by the fitted model. Table 5 presents summary statistics for the expected inflation rates for the various horizons.

The results indicate that the average term structure of inflation expectations is monotonically increasing during the 2004–2014 sample period. The average one-year expected inflation rate is 1.80 percent, while the average 30-year expected inflation rate is 3.19 percent. The table also shows that there is time variation in expected inflation, although the variation is surprisingly small for longer horizons. In particular, the standard deviation of expected inflation ranges from 1.20 percent for the one-year horizon to less than 0.30 percent for horizons of ten years or longer. To illustrate the time variation in expected inflation more clearly, Figure 3 plots the expected inflation estimates for the one-, two-, five-, and ten-year horizons.

It is also interesting to contrast these market-implied forecasts of inflation with forecasts provided by major inflation surveys. As discussed by Ang, Bekaert, and Wei (2007), these surveys of inflation tend to be more accurate than those based on standard econometric models and are widely used by market practitioners. Furthermore, these inflation surveys have also been incorporated into a number of important academic studies of inflation such as Fama and Gibbons (1984), Chernov and Mueller (2012) and Haubrich, Pennachi, and Ritchken (2012).

We obtain inflation expectations from three surveys: the University of Michigan Survey of Consumers, the Philadelphia Federal Reserve Bank Survey of Professional Forecasters (SPF), and the Livingston Survey. The sample period for the forecasts matches that for the inflation swap data in the study. The Appen-
dix provides the background information and details about how these surveys are conducted.

Table 6 reports the average values of the various surveys during the sample period and the corresponding average values for the market-implied forecasts. These averages are computed using the prior month-end values for the months in which surveys are released. Thus, monthly averages are compared with monthly averages, quarterly averages with quarterly averages, etc. As shown, the average market-implied forecasts of inflation tend to be a little lower than the survey averages for shorter horizons. The market-implied forecasts, however, closely parallel those from the surveys for longer horizons. While it would be interesting to compare the relative accuracy of the market-implied and survey forecasts, our sample is too short to do this rigorously.

6.4 Inflation Volatility

Table 7 reports summary statistics for the estimated volatility of the annualized inflation rate for horizons ranging from one year to 30 years. The average inflation volatility estimates are surprisingly similar across the horizons, with values on the order of 1.30 to 1.50 percent. Inflation volatility itself, however, is much more volatile for shorter horizons.

7. DEFLATION RISK

We turn now to the central issue of measuring the risk of deflation implied by market prices and studying the properties of deflation risk. First, we present descriptive statistics for the implied deflation risk. We then examine how the market prices the tail risk of deflation and contrast the results with those found in other markets.

7.1 How Large is the Risk of Deflation?

Having solved for the inflation distribution, we can directly compute the probability that the average realized inflation rate over a specific horizon is less than zero, which represents the risk of deflation. Table 8 provides summary statistics for the estimated probabilities of deflation over the various horizons. To provide some additional perspective, Figure 4 graphs the time series of probabilities of a deflation over one-, two-, five-, and ten-year horizons.

As shown, the market places a surprisingly large weight on the possibility that deflation may occur over extended horizons. In particular, the average probability that the realized inflation rate will be less than or equal to zero is 13.73 percent for a one-year horizon, 11.36 percent for a two-year horizon, 6.34
percent for a five-year horizon, 5.23 percent for a ten-year horizon, and 2.77 percent for longer horizons.

What is perhaps more striking is that the probability of deflation varies significantly over time and reaches relatively high levels during the sample period. For example, the probability of deflation reaches a value of 41.73 percent for a one-year horizon, 34.00 percent for a two-year horizon, and 22.23 percent for a five-year horizon. At other times, the market assesses the probability of deflation at any horizon to be less than one percent. This variation in the probability of deflation is due not only to changes in expected inflation, but also to changes in the volatility of inflation.

These probabilities are broadly consistent with the historical record on inflation in the US. For example, based on the historical inflation rates from 1800 to 2012, the US has experienced deflation over a one-year horizon 65 times, which represents a frequency of 30.5 percent. Considering only nonoverlapping periods, the US has experienced a two-year deflation 41 times, a five-year deflation 19 times, a ten-year deflation 11 times, and a 30-year deflation three times. These translate into frequencies of 24.0 percent, 14.4 percent, 10.6 percent, and 3.1 percent, respectively.\(^{10}\)

Figure 4 also shows that the deflation probabilities for the shorter horizons have occasional jumps upward. These jumps tend to occur around major financial events such as those associated with the European Debt Crisis. For example, the Eurozone experienced major turmoil during April and May of 2010 as concerns about the ongoing solvency of Portugal, Italy, Ireland, Greece, and Spain become more urgent and a number of bailout plans were put into place. Spain’s debt was first downgraded by Fitch on May 29, 2010. The one-year deflation probabilities increases dramatically during this period. In addition, the one-year deflation probability spikes again in early August of 2011, coinciding with the downgrade of US Treasury debt by Standard and Poors. We will explore the link between deflation risk and major financial risk more formally later in the paper.

Although not shown, we also calculate the partial moment in which we take the expected value of inflation conditional on the inflation rate being less than or equal to zero. This partial moment provides a measure of the severity of a deflation, conditional on deflation occurring over some horizon. For example, finding that this partial moment was only slightly negative would argue that a deflationary episode was likely to be less severe, while the opposite would be true for a more negative value of this partial moment. The results indicate that

\(^{10}\)Historical inflation rates are tabulated by Sahr (2012), oregonstate.edu/cla/polisci/faculty-research/sahr/infcf17742007.pdf. More recent inflation rates are reported by the Bureau of Labor Statistics.
the expected severity of a deflation is typically substantial with these conditional moments of $-0.72$ percent for a one-year deflation, $-0.61$ for a ten-year horizon, and $-0.53$ percent for a 30-year horizon. Note that a deflation of $-0.50$ percent per year would translate into a decline in the price level of $4.88$ percent over a ten-year period, $9.54$ percent over a 20-year period, and $13.96$ percent over a 30-year period. These would represent protracted deflationary episodes comparable to some of those experienced historically in the US.

7.2 Pricing Deflation Tail Risk

Although we have solved for the inflation risk premium embedded in inflation swaps earlier in the paper, it is also interesting to examine how the market prices the risk that the tail event of a deflation occurs. This analysis can provide insight into how financial market participants view the risk of events that may happen infrequently, but which may have catastrophic implications.

A number of these types of tail risks have been previously studied in the literature. For example, researchers have investigated the pricing of catastrophic insurance losses such as those caused by hurricanes or earthquakes. Froot (2001) finds that the ratio of insurance premia to expected losses in the market for catastrophic reinsurance is two or more during the 1989 to 1998 period. Lane and Mahul (2008) estimate that the pricing of catastrophic risk in a sample of 250 catastrophe bonds is about $2.69$ times the actual expected loss over the long term. Garmaise and Moskowitz (2009) and Ibragimov, Jaffee, and Walden (2011) offer both empirical and theoretical evidence that the extreme left tail catastrophic risk can be significantly priced in the market.

The default of a corporate bond is also an example of an event that is relatively rare for a specific firm, but which would result in an extremely negative outcome for bondholders of the defaulting firm. The pricing of default risk has been considered in many recent papers. For example, Giesecke, Longstaff, Schaefer, and Strebulaev (2011) study the pricing of corporate bond default risk and find that the ratio of corporate credit spreads to their actuarial expected loss is $2.04$ over a 150-year period. Similarly, Driessen (2005) and Berndt, Duffie, Douglas, Ferguson, and Schranz (2005) estimate ratios using data for recent periods that range in value from about $1.80$ to $2.80$.

Following along the lines of this literature, we solve for the ratio of the risk-neutral probability of deflation to the objective probability of deflation. This ratio provides a simple measure of how the market prices the tail risk of deflation and has the advantage of being directly comparable to the ratios discussed above. To avoid computing ratios when the denominator is near zero, we only compute the ratio when the objective probability of deflation is in excess of $0.01$ percent. We use a similar procedure to solve for the ratios of the risk-neutral to objective
probabilities for the events that the realized inflation rate is less than −1.00 percent and −2.00 percent, respectively.

Before presenting the results, it may be useful to first review why there could be significant differences between tail probabilities under the objective and risk-neutral measures even though inflation is normally distributed under both. First, the means of the two distributions may differ because of the inflation risk premium embedded in inflation derivative prices. Second, and perhaps most importantly, the variance of the inflation distribution can differ significantly across the two measures. The reason for this is that the speed at which expected inflation variable reverts back to its long-run mean is very different across the two measures. The speed of mean reversion parameter, however, can have a large impact on variance of inflation over any finite horizon. The combination of the differences in means and variances between the objective and risk-neutral distributions can map into significant differences in both the left and right tail probabilities.

Table 9 presents summary statistics for the ratios for the various horizons. As shown, the mean ratios for deflation range generally range from about 1.10 to as high as 1.70. On average, the ratios for the event that the inflation rate is less than −1.00 percent are often in excess of 1.30, particularly for maturities of up to ten years. Similarly, many of the average ratios for the event that the inflation rate is less than −2.00 are in the range of 1.50 to 2.30. These values are in the same ballpark as those for the different types of tail risk discussed above. These ratios all indicate that the market is deeply concerned about economic tail risks that may be difficult to diversify or may be strongly systematic in nature.

8. WHAT DRIVES DEFLATION RISK?

A key advantage of our approach is that by extracting the market’s assessment of the objective probability of deflation, we can then examine the relation between these probabilities and other financial and macroeconomic factors. In particular, we can study the relation between the tail risk of deflation and other types of tail risk that may be present in the markets.

In doing this, we focus on four broad categories of tail risk that have been extensively discussed in the literature. Specifically, we consider the links between deflation risk and systemic financial system risk, collateral revaluation risk, sovereign default risk, and business cycle risk.

The link between systemic risk in the financial system and major economic crises is well established in many important papers including Bernanke (1983), Bernanke, Gertler, and Gilchrist (1996), and others. Systemic risk in the financial
system is widely viewed as having played a central role in the recent global financial crisis and represents a motivating force behind major regulatory reforms such as the Dodd-Frank Act. We use several measures of systemic risk in the analysis.

First, we use the spread between the three-month Libor rate and the overnight index swap (OIS) rate. This spread is widely followed in the financial markets and provides a measure of the difference between short-term financing costs of the banking sector versus the fed funds rate. A sharp increase in this spread is typically viewed as a signal of short-term systemic stress within the financial sector.

Second, we use the five-year swap spread as a measure of the systemic credit and liquidity stresses on the financial system. As discussed by Duffie and Singleton (1997), Liu, Longstaff, and Mandell (2006), and others, the swap spread also reflects differences in the relative liquidity and credit risk of the financial sector and the Treasury. We obtain five-year swap spread data from the Bloomberg system.

We also considered a number of other measures of systemic risk such as the average CDS spread for both major US and non-US banks and financial firms. These measures, however, were highly correlated with the other measures such as swap spreads and provided little incremental information.

Recent economic theory has emphasized the role that the value of collateral plays in propagating economic downturns. Key examples include Kiyotaki and Moore (1997) who show that declines in asset values can lead to contractions in the amount of credit available in the market which, in turn, can lead to further rounds of declines in asset values. Bernanke and Gertler (1995) describe similar interactions between declines in the value of assets that serve as collateral and severe economic downturns. Collateral revaluation risk, or the risk of a broad decline in the market value of leveraged assets, played a major role in the Great Depression as the sharp declines in the values of stock and corporate bonds triggered waves of defaults among both speculators and banks. A similar mechanism was present in the recent financial crisis as sharp declines in real estate values led to massive defaults by “underwater” mortgagors. In the context of this study, we explore the relation between deflation probabilities and valuations in several major asset classes that may represent important sources of collateral in the credit markets: stocks and bonds. In addition, we also include measures of the volatility of these asset classes since these measures provide information about the risk that large downward revaluations in these forms of collateral may occur.

The first of these proxies for collateral revaluation risk is the VIX index of
implied volatility for options on the S&P 500. This well-known index is often termed the “fear index” in the financial press since it reflects the market’s assessment of the risk of a large downward movement in the stock market. We collect VIX data from the Bloomberg system.

The second measure is simply the time series of returns on the value-weighted CRSP stock index. This return series reflects changes in the value of one of the largest potential sources of collateral in the macroeconomy.

The third measure is the Markit North American Investment Grade Index of CDS spreads. This index consists of an average of the quoted CDS spreads for a liquid basket of 125 firms from a wide variety of industries and sectors and is a widely-followed measure of credit spreads in the financial markets. Variation in the CDX index over time reflects changes in the market’s assessment of default risk in the economy as well as the pricing of credit risk. We collect data on the CDX index from the Bloomberg system.

Another major type of economic tail risk stems from the risk that a sovereign defaults on its debt. As documented by Reinhart and Rogoff (2009) and many others, sovereign defaults tend to be associated with severe economic crisis scenarios. As a measure of the tail risk of a major sovereign default, we include in the analysis the time series of sovereign CDS spreads on the US Treasury and Germany. Ang and Longstaff (2013) show that the US CDS spread reflects variation in the valuation of major sources of tax revenue for the US such as capital gains on stocks and bonds. Similarly, for the CDS spread for Germany. This data is also obtained from the Bloomberg system.

Finally, to capture the effect of traditional types of business cycle risk or economic downturn risk, we also include a number of key macroeconomic variables that can be measured at a monthly frequency. In particular, we include the monthly percentage change in industrial production as reported by the Bureau of Economic Analysis, the monthly change in the national unemployment rate as reported by the Bureau of Labor Statistics, and the change in the Consumer Confidence Index reported by the University of Michigan. The link between the business cycle and its effects on output and employment are well established in the macroeconomic literature and forms the basis of many classical theories including the Phillips curve.

In examining the relation between these measures of tail risk and deflation risk, we will use the simple approach of regressing monthly changes in deflation probabilities on monthly changes in these measures. In doing this, it is important to note that while these variables were chosen as a measure of a specific type of tail risk, most of these variables may actually reflect more than one type of tail risk. Thus, the effects of the variables in the regression should be interpreted
carefully since the different types of tail risk need not be mutually exclusive.

Table 10 presents summary statistics for the regression results. These results provide a number of intriguing insights into the nature of deflation risk.

First, the results illustrate that short-term deflation risk tends to be driven by a different set of factors than is longer-term deflation risk. For example, Table 10 shows that many of the financial variables in the regression are significantly related to changes in deflation risk for horizons up to five years, but few are significant for longer horizons. In particular, of the two measures of systemic financial risk, one or both are significantly for the one- and two-year horizons. The positive signs for the coefficients implies that deflation risk increases as these measures of stress in the financial markets increase. This is consistent with an economic scenario in which stress in the financial sector leads to an increase in the perceived risk of an adverse macroeconomic shock in which price levels decline. Furthermore, the return on the stock market and changes in the VIX index are both highly significant for many of the shorter-term horizons. The positive sign for the stock index returns is consistent with the notion that deflation fears diminish as asset valuations increase. While the negative sign for changes in the VIX index is less intuitive, it is important to recognize that the VIX index and the stock market are strongly negatively correlated and this could influence the partial effects captured in the regression. Taken together, these results suggest that short-term deflation risk is strongly linked to conditions in the financial markets.

For intermediate horizons, deflation risk seems to be linked more closely to structural factors such as credit risk in the corporate and sovereign sectors. Specifically, changes in the sovereign CDS spread for Germany are significant (at least at the ten-percent level) for the two-, three-, seven-, and 12-year horizons. The positive sign of these significant coefficients is consistent with the view that the ongoing sovereign credit crisis within Europe raised deflationary fears in the US. This interpretation is supported by the result that the US Treasury CDS spread is significantly negative for the two- and three-year horizons, suggesting that it may be the contrast between the two sovereign spreads that matters. In other words, it is the portion of the CDS spread that is unique to Germany (presumably the risk of the breakup of the Eurozone) that is the most related to changes in US deflation risk. The results in Table 10 also suggest that concerns about longer-term credit risk in the corporate sector are also related to deflation fears. Specifically, the CDX credit index is significant (at the ten-percent level) for the ten-, 12-, and 30-year horizons. These results also underscore the importance of understanding the role that the credit sector plays in economic downturns such as the recent financial crisis that began in the subprime structured credit markets.
For long-term horizons of ten to 30 years, deflation risk seems more related to fundamental macroeconomic factors. In particular, changes in industrial production are significant for the 20- and 30-year horizons. Changes in unemployment are positive and significant for the five-, 12-, and 30-year horizons. The positive sign of these coefficients is intuitive since it indicates that deflation risk increases as the unemployment rate increases. This is also consistent with classical macroeconomic theory about the relation between price levels and unemployment such as the Phillips curve. Changes in consumer confidence are significant for both the 20- and 30-year horizons (as well as for the one-year horizon). The negative sign for these significant coefficients is also intuitive since it implies that deflationary fears dissipate as confidence increase.

Second, the overall adjusted $R^2$s from the regressions are surprisingly high. In particular, the adjusted $R^2$s for the two- through five-year horizons as well as the 30-year horizon are in excess of 40 percent. Several of the other adjusted $R^2$s are in excess of 20 percent. These high $R^2$s suggest that the tail risk of deflation is strongly correlated with tail risks in other sectors of the macroeconomy.

9. INFLATION RISK

Although the focus of this paper is on deflation risk, it is straightforward to extend the analysis to other aspects of the distribution of inflation. As one last illustration of this, we compute the probabilities that the inflation rate exceeds values of four, five, and six percent using the techniques described earlier. Table 11 reports summary statistics for these probabilities.

As shown, the market-implied probabilities of experiencing significant inflation are relatively large. Specifically, the average probability of inflation exceeding four percent ranges from 5.14 percent for the one-year horizon to nearly 20 percent for the 30-year horizon. Most of the average probabilities are in excess of 10 percent. Figure 5 plots the time series of probabilities that inflation exceeds four percent for several horizons. These plots also show that the probability of inflation in the long run is substantially higher than in the short turn. This is the opposite of the situation for deflation risk which tends to be higher in the short run.

Table 11 also shows that the average probability of an inflation rate in excess of five percent is substantial. For example, the average probability of inflation in excess of five percent over a 30 year period is 6.30 percent. Note that an average inflation rate of five percent or more over a period of decades would rival any inflationary scenario experienced by the US during the past 200 years. Finally, the results show that the market anticipates that there is more than a
one-percent probability of inflation averaging more than six percent over the next several decades.

As we did earlier for deflation tail risk, we can also examine the pricing of inflation tail risk by computing the ratio of the probability of inflation under the risk-neutral measure to the corresponding probability under the actual measure. Table 12 provides summary statistics for the ratios.

As illustrated, inflation tail risk is priced at all but the longest horizon. The magnitude of the inflation risk premium, however, is significantly smaller than is the case for deflation tail risk. The mean ratios for the event of inflation exceeding four percent are only slightly above one. Similarly, the mean ratios for the events of inflation exceeding five and six percent are are generally in the range of 1.00 to 1.40.

Consumption disaster models can replicate the key facts about nominal bond return predictability provided that inflation jumps in a disaster (see Gabaix (2012)). As a result of these inflation jumps in disaster states, the risk-neutral probability of a large inflation is much higher than the actual probability, and the nominal bond risk premium increases as a result. Our results point in the opposite direction. We actually find direct evidence from inflation options that market participants are pricing in large deflation in disaster states, because the risk-neutral probability of a deflation is larger than the actual probability. This actually makes nominal bonds less risky because they provide a hedge against large consumption disasters.

10. CONCLUSION

We solve for the objective distribution of inflation using the market prices of inflation swap and option contracts and study the nature of deflation risk. We find that the market-implied probabilities of deflation are substantial, even though the expected inflation rate is roughly 2.75 to 3.25 percent for horizons of up to 30 years. We show that deflation risk is priced by the market in a manner similar to that of other major types of tail risk such as catastrophic insurance losses or corporate bond defaults. By embedding a deflation floor into newly issued TIPS, the Treasury insures bondholders against deflation. Our findings imply that the Treasury receives a generous insurance premium in return. In contrast, the market appears less concerned about inflation tail risk.

In theory, economic tail risks such as deflation may be related to other financial and macroeconomic tail risks. We study the relation between deflation risk and a number of measures of systemic financial risk, collateral revaluation risk, sovereign credit risk, and business cycle risk. We find that these measures
of tail risk have sizable explanatory power for the variation in deflation risk. We also find that the nature of deflation risk varies with horizon. In particular, short-term deflation risk is strongly related to financial variables, while intermediate-term deflation risk is more related to structural measures such as corporate and sovereign credit risk. Long-term deflation risk is driven more by macroeconomic factors. These results support the view that the risk of economic shocks severe enough to result in deflation is fundamentally related to the risk of major shocks in the financial markets both locally and globally.
REFERENCES


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A.1 The Distribution of the Price Level.

In this section, we derive the distribution of $I_T$ under the actual or objective measure $P$. We note, however, that the distribution of $I_T$ under the risk-neutral measure can be obtained in exactly the same way.

From Equation (1), the relative price level index at time $T$ can be expressed as

$$I_T = \exp \left( \int_0^T X_s \, ds - \frac{1}{2} \sigma^2 T + \sigma \int_0^T dZ_t(s) \right). \quad (A1)$$

Thus,

$$\ln I_T = \int_0^T X_t \, dt - \frac{1}{2} \sigma^2 T + \sigma \int_0^T dZ_t(s). \quad (A2)$$

To solve for the distribution of $\ln I_T$, we first solve for the distribution of $\int_0^T X_t \, dt$. Solving the stochastic differential equation in Equation (3) for $Y_t$ gives,

$$Y_t = Y e^{-\beta t} + \left( \frac{\mu}{\beta} \right) \left( 1 - e^{-\beta t} \right) + s e^{-\beta t} \int_0^t e^{\beta u} \, dZ_Y(u). \quad (A3)$$

Likewise, solving for $X_t$ gives

$$X_t = X e^{-\kappa t} + \kappa e^{-\kappa t} \int_0^t e^{\kappa u} \, Y_u \, du + \eta e^{-\kappa t} \int_0^t e^{\kappa u} \, dZ_X(u). \quad (A4)$$

Substituting Equation (A3) into the above equation, interchanging the order of integration, and evaluating terms gives the following expression for $X_t$,

$$X_t = X e^{-\kappa t} + \left( \frac{\mu}{\beta} \right) \left( 1 - e^{-\kappa t} \right) + \frac{\kappa}{\kappa - \beta} \frac{\kappa}{\kappa - \beta} \left( Y - \frac{\mu}{\beta} \right) \left( e^{-\beta t} - e^{-\kappa t} \right)$$

$$+ s \kappa / (\kappa - \beta) \int_0^t e^{-\beta u} \, e^{\beta u} - e^{-\kappa t} e^{\kappa u} \, dZ_Y(u) + \eta e^{-\kappa t} \int_0^t e^{\kappa u} \, dZ_X(u). \quad (A5)$$

Taking the integral of $X_t$, interchanging the order of integration, and evaluating terms gives
\[
\int_0^T X_t \, dt = \mu T/\beta + (X - \mu/\beta)(1 - e^{-\kappa T})/\kappa \\
+ (\kappa/(\kappa - \beta))(Y - \mu/\beta)((1 - e^{-\beta T})/\beta - (1 - e^{-\kappa T})/\kappa) \\
+ s\kappa/(\kappa - \beta) \int_0^T (1 - e^{-\beta(T-u)})/\beta - (1 - e^{-\kappa(T-u)})/\kappa \, dZ_Y(u) \\
+ \eta \int_0^T (1 - e^{-\kappa(T-u)})/\kappa \, dZ_X(u).
\]  
(A6)

Thus, \( \int_0^T X_t \, dt \) is a normally distributed random variable. This result, in conjunction with Equation (A2), implies that \( \ln I_T \) is normally distributed with mean

\[
\mu T/\beta + (1/\kappa)(X - \mu/\beta)(1 - e^{-\kappa T}) \\
+ (\kappa/(\kappa - \beta))(Y - \mu/\beta)((1 - e^{-\beta T})/\beta - (1 - e^{-\kappa T})/\kappa) - \sigma^2 T/2, \quad (A7)
\]

and variance

\[
- \frac{s^2 \kappa^2}{(\kappa - \beta)^2} \left( \frac{1}{\beta^2} \left( T - \frac{2}{\beta} (1 - e^{-\beta T}) + \frac{1}{2\beta} (1 - e^{-2\beta T}) \right) \\
- \frac{2}{\beta \kappa} (T - \frac{1}{\beta} (1 - e^{-\beta T}) - \frac{1}{\kappa} (1 - e^{-\kappa T}) + \frac{1}{\beta + \kappa} (1 - e^{-(\beta + \kappa) T})) \\
+ \frac{1}{\kappa^2} (T - \frac{2}{\kappa} (1 - e^{-\kappa T}) + \frac{1}{2\kappa} (1 - e^{-2\kappa T})) \right) \\
+ \frac{\eta^2}{\kappa^2} \left( T - \frac{2}{\kappa} (1 - e^{-\kappa T}) + \frac{1}{2\kappa} (1 - e^{-2\kappa T}) \right) + \sigma^2 T. \quad (A8)
\]

Under the risk-neutral measure, \( \ln I_T \) is also normally distributed with a mean and variance given by the expressions in Equations (A7) and (A8), but where the parameters \( \mu \) and \( \kappa \) are replaced with \( \alpha \) and \( \lambda \), respectively.

A.2 The Inflation Swap Rate.

The cash flow associated with a zero-coupon inflation swap at time \( T \) is simply \( I_T - F(X,Y,T) \) where \( F(X,Y,T) \) is the inflation swap price at the initiation of the contract at time zero. Since the present value of the inflation swap is zero at inception we have,

\[
E^Q \left[ \exp \left( -\int_0^T r_s \, ds \right) (I_T - F(X,Y,T)) \right] = 0, \quad (A9)
\]
where the expectation is taken with respect to the risk-neutral distribution (denoted by a $Q$). Substituting in for $r_t$ and $I_T$ gives,

$$
E^Q \left[ \exp \left( -\int_0^T R_s \, ds \right) \right] 
\left[ E^Q \left[ \exp \left( -\int_0^T X_s \, ds \right) \exp \left( \int_0^T X_s \, ds - \frac{1}{2}\sigma^2 T + \int_0^T dZ_I(s) \right) \right] 
- E^Q \left[ \exp \left( -\int_0^T X_s \, ds \right) F(X,Y,T) \right] \right] = 0,
$$

(A10)

which implies

$$
F(X,Y,T) = \frac{E^Q \left[ \exp \left( -\frac{1}{2}\sigma^2 T + \sigma \int_0^T dZ_I(s) \right) \right]}{E^Q \left[ \exp \left( -\int_0^T X_s \, ds \right) \right]},
$$

(A11)

$$
= \frac{1}{E^Q \left[ \exp \left( -\int_0^T X_s \, ds \right) \right]}.
$$

(A12)

Let $H(X,Y,\tau)$ denote the value of the expectation $E^Q[\exp(-\int_t^T X_s \, ds)]$, where $\tau = T - t$. Standard results imply that this expectation satisfies the partial differential equation

$$
\frac{1}{2}\eta^2 H_{XX} + \frac{1}{2}s^2 H_{YY} + \lambda(Y - X)H_X + (\alpha - \beta Y)H_Y - XH = H_\tau,
$$

(A13)

subject to the terminal condition $H(X,Y,0) = 1$. We conjecture a solution of the form $H(X,Y,\tau) = \exp(A(\tau) + B(\tau)X + C(\tau)Y)$. Taking derivatives of this expression and substituting into Equation (A13) results in a system of three linear first-order ordinary differential equations for the horizon-dependent functions $A(\tau)$, $B(\tau)$, and $C(\tau)$,

$$
B' + \lambda B = -1,
$$

(A14)

$$
C' + \beta C = \lambda B,
$$

(A15)

$$
A' = \frac{1}{2}\eta^2 B^2 + \frac{1}{2}s^2 C^2 + \alpha C.
$$

(A16)

These three equations are readily solved by the use of an integrating factor and direct integration. We substitute these solutions into the expression for $H(X,Y,\tau)$,
and then substitute $H(X,Y,\tau)$ into Equation (A12). Finally, evaluating as of time zero ($\tau = T$) gives an expression for the inflation swap price that can be written in the form of Equation (7) (as part of this, we rewrite the solution for $A(\tau)$ in Equation (A16) in the form of the expression $A(T) + V(T)$ in Equations (8) and (11)).

A.3 Inflation Option Prices.

Let $C(X,Y,T)$ denote the price at time zero of a European call option on the price level at time $T$ with strike $K$. The cash flow at the option expiration date is $\max(0, I_T - (1 + K)T)$. The present value of this cash flow can be expressed as

$$E^Q \left[ \exp \left( -\int_0^T r_s \, ds \right) \max(0, I_T - (1 + K)T) \right],$$

which can be written as

$$E^Q \left[ \exp \left( -\int_0^T R_s \, ds \right) \right] E^Q \left[ \exp \left( -\int_0^T X_s \, ds \right) \max(0, I_T - (1 + K)T) \right],$$

after substituting in for $r_t$. Let $N(I,X,Y,\tau)$ denote the value of the expectation $E^Q[\exp(-\int_0^T X_s \, ds) \max(0, I_T - (1 + K)T)]$, where for clarity we show the explicit functional dependence of $N(I,X,Y,\tau)$ on $I$. The value of $N(I,X,Y,\tau)$ satisfies the following partial differential equation,

$$\frac{1}{2} \sigma^2 I^2 N_{II} + \frac{1}{2} \eta^2 N_{XX} + \frac{1}{2} s^2 N_{YY} + XIN_I + \lambda(Y - X)N_X + (\alpha - \beta Y)N_Y - XN = N_{\tau},$$

subject to the terminal condition $N(I,X,Y,0) = \max(0, I_T - (1 + K)T)$. We conjecture that the solution is of the form

$$N(I,X,Y,\tau) = H(X,Y,\tau) M(I,X,Y,\tau).$$

Substituting this expression into the partial differential equation in Equation (A19), recognizing that $H(X,Y,\tau)$ satisfies Equation (A13), and simplifying gives
\[
\frac{1}{2} \sigma^2 I^2 M_{II} + \frac{1}{2} \eta^2 M_{XX} + \frac{1}{2} s^2 M_{YY} + XIM_I + (\lambda(Y - X) + \eta^2 B(\tau))M_X + (\alpha - \beta Y + s^2 C(\tau))M_Y = M_T. \tag{A21}
\]

This implies that \( M(I, X, Y, \tau) \) can be expressed as

\[
M(I, X, Y, \tau) = E^{Q^*} \left[ \max(0, I_T - (1 + K)^T) \right], \tag{A22}
\]

where the expectation is taken with respect to the density of \( I_T \) implied by the dynamics (we denote this measure by \( Q^* \)),

\[
dI = X_I dt + \sigma I \, dZ_I, \tag{A23}
\]
\[
dX = (\lambda (Y - X) + \eta^2 B(\tau)) \, dt + \eta \, dZ_X, \tag{A24}
\]
\[
dY = (\alpha - \beta Y + s^2 C(\tau)) \, dt + s \, dZ_Y. \tag{A25}
\]

Since

\[
D(T) = E^{Q} \left[ \exp \left( -\int_0^T r_s \, ds \right) \right], \tag{A26}
\]
\[
= E^{Q} \left[ \exp \left( -\int_0^T R_s \, ds \right) \right] H(X, Y, \tau), \tag{A27}
\]

combining these results implies

\[
C(X, Y, T) = D(T) \, E^{Q^*} \left[ \max(0, I_T - (1 + K)^T) \right]. \tag{A28}
\]

Note that under this measure, the expected value of the price level equals the inflation swap price \( F \). This follows since the cash flow from an inflation swap at time \( T \) is \( I_T - F \). Under the \( Q^* \) measure, however, the present value of this cash flow is given by \( D(T)E^{Q^*} [I_T - F] \). Since the initial value of the inflation swap contract is zero, this implies \( E^{Q^*} [I_T] = F \).

Following exactly the same line of analysis as in Section A.1, and using the result immediately above, it is easily shown that \( \ln I_T \) is normally distributed under measure \( Q^* \) with mean \( \ln F - V(T) - \frac{1}{2} \sigma^2 T \) and variance \( 2V(T) + \sigma^2 T \), where \( V(T) \) is as defined in Equation (11).
We can now solve for the expectation in Equation (A28) by simply integrating the payoff with respect to the density of $\ln I_T$ under measure $Q^*$. The resulting solution is closely related to the Black-Scholes equation and is given in Equation (12).

**A.4 The Conditional Moments.**

Integrating the dynamics for $X$ and $Y$ under the $P$ measure as given in Section A.1 results in the following expressions for the conditional means and variances

\[
\begin{align*}
\mu_Y &= Y_t e^{-\beta \Delta t} + (\mu / \beta) (1 - e^{-\beta \Delta t}), \\
\mu_X &= X_t e^{-\kappa \Delta t} + (\mu / \beta) (1 - e^{-\kappa \Delta t}) \\
&\quad + \frac{\kappa}{\kappa - \beta} (Y_t - \mu / \beta)(e^{-\beta \Delta t} - e^{-\kappa \Delta t}), \\
\sigma_Y &= \frac{s^2}{2\beta} (1 - e^{-2\beta \Delta t}), \\
\sigma_X &= \frac{\kappa^2 s^2}{(\kappa - \beta)^2} \left( \frac{1}{2\beta} (1 - e^{-2\beta \Delta t}) - \frac{2}{\beta + \kappa} (1 - e^{-(\beta + \kappa) \Delta t}) \\
&\quad + \frac{1}{2\kappa} (1 - e^{-2\kappa \Delta t}) \right) + \frac{\eta^2}{2\kappa} (1 - e^{-2\kappa \Delta t}), \\
\sigma_{XY} &= \frac{\kappa s^2}{\kappa - \beta} \left( \frac{1}{2\beta} (1 - e^{-2\beta \Delta t}) - \frac{1}{\beta + \kappa} (1 - e^{-(\beta + \kappa) \Delta t}) \right). 
\end{align*}
\]

**A.5 The Inflation Surveys.**

The data from the University of Michigan Survey of Consumers consist of one- and five-year ahead inflation forecasts. The series is released at monthly frequency and reports the median expected price change over the next twelve months and the next five years, respectively. A detailed description of how the survey is conducted is available at http://www.sca.isr.umich.edu/documents.php?c=i. In contrast to the Livingston survey and the Survey of Professional Forecasters, the participants in the University of Michigan Survey of Consumers are actual consumers (households) and not professionals. The time between the conduct of the survey and release is up to three weeks. The University of Michigan reports that a review of the estimates of inflation expectations indicated that for comparisons over time, the median, rather than the mean, may be a more reliable measure of the central tendency of the response distribution due to the changing influence of extreme responses. Therefore, we use the median survey forecasts throughout our analysis.
The Philadelphia Federal Reserve Bank Survey of Professional Forecasters is conducted on a quarterly basis. The questionnaires are sent to the participants at the end of January, April, July, and October, and the survey results are published in the middle of February, May, August, and November, for the first, second, third, and fourth quarter, respectively. In contrast to the Livingston survey, participants in the SPF forecast changes in the quarterly average CPI-U levels. A detailed description of how the survey is conducted is available at http://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professionalforecasters/spf-documentation.pdf.

The Livingston survey is conducted twice a year, in June and in December, usually in the middle of the month. Participants include economists from industry, government, and academia. The surveys taken in June consist of two annual average CPI forecasts: for the current year, and for the following year. The December surveys include three annual average forecasts: for the current year, for the next year, and for the year after. The participants forecast the non-seasonally-adjusted CPI level six and twelve months in the future. The survey also includes a forecast of the CPI ten years in the future. A detailed description of how the Livingston survey is conducted is available at: http://www.philadelphiafed.org/research-and-data/real-time-center/livingston-survey/livingston-documentation.pdf. For the Livingston surveys, there is a lag of up to four and three weeks between the time the survey and when the results are disseminated.
Figure 1. Inflation Densities. This figure plots the time series of inflation densities for horizons of one year (upper left), two years (upper right), five years (lower left), and ten years (lower right).
Figure 2. Inflation Risk Premia. This figure plots the time series of inflation risk premia for horizons of one year (upper left), five years (upper right), ten years (lower left), and 30 years (lower right).
Figure 3. Expected Inflation. This figure plots the time series of expected inflation for horizons of one year (upper left), five years (upper right), ten years (lower left), and 30 years (lower right).
Figure 4. Deflation Probabilities. This figure plots the time series of deflation probabilities for horizons of one year (upper left), two years (upper right), five years (lower left), and ten years (lower right).
Figure 5. Inflation Probabilities. This figure plots the time series of probabilities that inflation is greater than or equal to four percent for horizons of one year (upper left), two years (upper right), five years (lower left), and ten years (lower right).
### Table 1

**Summary Statistics for Inflation Swap Rates.** This table reports summary statistics for the inflation swap rates for the indicated maturities. Swap maturity is expressed in years. Inflation swap rates are expressed as percentages. The sample consists of daily observations for the period from July 23, 2004 to July 29, 2014.

<table>
<thead>
<tr>
<th>Swap Maturity</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.743</td>
<td>1.244</td>
<td>-4.545</td>
<td>1.862</td>
<td>3.802</td>
<td>2613</td>
</tr>
<tr>
<td>2</td>
<td>1.919</td>
<td>0.987</td>
<td>-3.605</td>
<td>1.992</td>
<td>3.460</td>
<td>2613</td>
</tr>
<tr>
<td>3</td>
<td>2.077</td>
<td>0.775</td>
<td>-2.047</td>
<td>2.141</td>
<td>3.351</td>
<td>2613</td>
</tr>
<tr>
<td>4</td>
<td>2.207</td>
<td>0.626</td>
<td>-1.228</td>
<td>2.268</td>
<td>3.342</td>
<td>2613</td>
</tr>
<tr>
<td>5</td>
<td>2.314</td>
<td>0.525</td>
<td>-0.570</td>
<td>2.377</td>
<td>3.310</td>
<td>2613</td>
</tr>
<tr>
<td>6</td>
<td>2.394</td>
<td>0.450</td>
<td>-0.080</td>
<td>2.468</td>
<td>3.310</td>
<td>2613</td>
</tr>
<tr>
<td>7</td>
<td>2.465</td>
<td>0.389</td>
<td>0.402</td>
<td>2.530</td>
<td>3.229</td>
<td>2613</td>
</tr>
<tr>
<td>8</td>
<td>2.522</td>
<td>0.344</td>
<td>0.640</td>
<td>2.580</td>
<td>3.195</td>
<td>2613</td>
</tr>
<tr>
<td>9</td>
<td>2.570</td>
<td>0.304</td>
<td>0.904</td>
<td>2.622</td>
<td>3.135</td>
<td>2613</td>
</tr>
<tr>
<td>10</td>
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<td>0.272</td>
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<td>2.664</td>
<td>3.145</td>
<td>2613</td>
</tr>
<tr>
<td>12</td>
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<td>0.255</td>
<td>1.280</td>
<td>2.713</td>
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</tr>
<tr>
<td>15</td>
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<td>0.256</td>
<td>1.161</td>
<td>2.778</td>
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</tr>
<tr>
<td>20</td>
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<td>1.070</td>
<td>2.845</td>
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<td>2613</td>
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<tr>
<td>25</td>
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<td>0.277</td>
<td>1.211</td>
<td>2.887</td>
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<tr>
<td>30</td>
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<td>3.500</td>
<td>2613</td>
</tr>
<tr>
<td>35</td>
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<td>2.353</td>
<td>2.809</td>
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</tr>
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<td>45</td>
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<td>1.465</td>
<td>2.821</td>
<td>3.500</td>
<td>1649</td>
</tr>
<tr>
<td>55</td>
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<td>0.165</td>
<td>2.386</td>
<td>2.861</td>
<td>3.287</td>
<td>1023</td>
</tr>
</tbody>
</table>
Table 2

Summary Statistics for Inflation Caps and Floors. This table reports the average values for inflation caps and floors for the indicated maturities and strikes. The average values are expressed in terms of basis points per $100 notional. Option Maturity is expressed in years. Ave. denotes the average number of caps and floors available each day from which the risk-neutral density of inflation is estimated. N denotes the number of days for which the risk-neutral density of inflation is estimated. The sample consists of daily observations for the period from October 5, 2009 to July 29, 2014.

<table>
<thead>
<tr>
<th>Option Maturity</th>
<th>Average Floor Value by Strike</th>
<th>Average Cap Value by Strike</th>
<th>Ave.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>−2</td>
<td>−1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>8</td>
<td>17</td>
<td>38</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>19</td>
<td>33</td>
<td>67</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>22</td>
<td>37</td>
<td>76</td>
</tr>
<tr>
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<td>23</td>
<td>33</td>
<td>50</td>
<td>103</td>
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<td>34</td>
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<td>115</td>
</tr>
<tr>
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</tr>
<tr>
<td>12</td>
<td>17</td>
<td>28</td>
<td>58</td>
<td>136</td>
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<tr>
<td>15</td>
<td>16</td>
<td>27</td>
<td>55</td>
<td>136</td>
</tr>
<tr>
<td>20</td>
<td>15</td>
<td>25</td>
<td>54</td>
<td>136</td>
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<tr>
<td>30</td>
<td>10</td>
<td>20</td>
<td>52</td>
<td>154</td>
</tr>
</tbody>
</table>
Table 3

Maximum Likelihood Estimation of the Inflation Swap Model. This table reports the maximum likelihood estimates of the parameters of the inflation swap model along with their asymptotic standard errors. The model is estimated using daily inflation swap prices for the period from July 23, 2004 to July 29, 2014.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.963868</td>
<td>0.324027</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.042007</td>
<td>0.000625</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.006743</td>
<td>0.002709</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.026974</td>
<td>0.111689</td>
</tr>
<tr>
<td>$s$</td>
<td>0.005720</td>
<td>0.000008</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.866571</td>
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<tr>
<td>$\alpha$</td>
<td>0.001680</td>
<td>0.000001</td>
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<tr>
<td>$v_1$</td>
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</tr>
<tr>
<td>$v_2$</td>
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<tr>
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</tr>
<tr>
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<tr>
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</tr>
<tr>
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</tr>
<tr>
<td>$v_8$</td>
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</tr>
<tr>
<td>$v_9$</td>
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</tr>
<tr>
<td>$v_{25}$</td>
<td>0.00000047</td>
<td>0.00000016</td>
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</table>
Table 4

Summary Statistics for Inflation Risk Premia. This table reports summary statistics for the estimated annualized inflation risk premia for the indicated horizons. Horizon is expressed in years. The inflation risk premia are measured in basis points. The inflation risk premia are estimated using the period from July 23, 2004 to July 29, 2014.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-5.45</td>
<td>4.58</td>
<td>-30.01</td>
<td>-4.80</td>
<td>4.46</td>
<td>2613</td>
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<tr>
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<td>-37.28</td>
<td>-7.01</td>
<td>3.81</td>
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<tr>
<td>3</td>
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<td>-35.50</td>
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<td>2613</td>
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<tr>
<td>4</td>
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<td>4.45</td>
<td>-30.84</td>
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<td>3.01</td>
<td>2613</td>
</tr>
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<td>-2.17</td>
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<td>-13.70</td>
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<td>5.92</td>
<td>2613</td>
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<td>9</td>
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<td>-10.99</td>
<td>2.19</td>
<td>6.58</td>
<td>2613</td>
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<td>10</td>
<td>2.79</td>
<td>2.08</td>
<td>-8.66</td>
<td>3.08</td>
<td>7.04</td>
<td>2613</td>
</tr>
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<td>12</td>
<td>3.72</td>
<td>1.74</td>
<td>-6.08</td>
<td>3.99</td>
<td>7.26</td>
<td>2613</td>
</tr>
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<td>15</td>
<td>2.91</td>
<td>1.40</td>
<td>-4.99</td>
<td>3.13</td>
<td>5.73</td>
<td>2613</td>
</tr>
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<td>20</td>
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<td>1.05</td>
<td>-9.47</td>
<td>-3.39</td>
<td>-1.45</td>
<td>2613</td>
</tr>
<tr>
<td>30</td>
<td>-30.20</td>
<td>0.67</td>
<td>-33.94</td>
<td>-30.10</td>
<td>-28.80</td>
<td>2613</td>
</tr>
</tbody>
</table>
Table 5

Summary Statistics for Expected Inflation. This table reports summary statistics for the expected inflation rate for the indicated horizons. Horizon is expressed in years. Expected inflation rates are expressed as percentages. The sample consists of daily observations for the period from July 23, 2004 to July 29, 2014.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.798</td>
<td>1.201</td>
<td>-4.263</td>
<td>1.914</td>
<td>3.763</td>
<td>2613</td>
</tr>
<tr>
<td>2</td>
<td>1.997</td>
<td>0.936</td>
<td>-3.232</td>
<td>2.065</td>
<td>3.421</td>
<td>2613</td>
</tr>
<tr>
<td>3</td>
<td>2.154</td>
<td>0.728</td>
<td>-1.692</td>
<td>2.215</td>
<td>3.326</td>
<td>2613</td>
</tr>
<tr>
<td>4</td>
<td>2.270</td>
<td>0.586</td>
<td>-0.930</td>
<td>2.326</td>
<td>3.324</td>
<td>2613</td>
</tr>
<tr>
<td>5</td>
<td>2.359</td>
<td>0.491</td>
<td>-0.322</td>
<td>2.422</td>
<td>3.288</td>
<td>2613</td>
</tr>
<tr>
<td>6</td>
<td>2.421</td>
<td>0.422</td>
<td>0.112</td>
<td>2.488</td>
<td>3.271</td>
<td>2613</td>
</tr>
<tr>
<td>7</td>
<td>2.474</td>
<td>0.365</td>
<td>0.572</td>
<td>2.536</td>
<td>3.181</td>
<td>2613</td>
</tr>
<tr>
<td>8</td>
<td>2.516</td>
<td>0.324</td>
<td>0.776</td>
<td>2.570</td>
<td>3.163</td>
<td>2613</td>
</tr>
<tr>
<td>9</td>
<td>2.551</td>
<td>0.286</td>
<td>0.960</td>
<td>2.600</td>
<td>3.104</td>
<td>2613</td>
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<tr>
<td>10</td>
<td>2.587</td>
<td>0.256</td>
<td>1.184</td>
<td>2.632</td>
<td>3.106</td>
<td>2613</td>
</tr>
<tr>
<td>12</td>
<td>2.634</td>
<td>0.243</td>
<td>1.299</td>
<td>2.672</td>
<td>3.113</td>
<td>2613</td>
</tr>
<tr>
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<td>2.710</td>
<td>0.247</td>
<td>1.177</td>
<td>2.747</td>
<td>3.256</td>
<td>2613</td>
</tr>
<tr>
<td>20</td>
<td>2.836</td>
<td>0.259</td>
<td>1.139</td>
<td>2.878</td>
<td>3.385</td>
<td>2613</td>
</tr>
<tr>
<td>25</td>
<td>2.992</td>
<td>0.271</td>
<td>1.387</td>
<td>3.034</td>
<td>3.532</td>
<td>2613</td>
</tr>
<tr>
<td>30</td>
<td>3.190</td>
<td>0.273</td>
<td>1.777</td>
<td>3.224</td>
<td>3.797</td>
<td>2613</td>
</tr>
</tbody>
</table>
**Comparison of Survey Forecasts with Market-Implied Forecasts.** This table reports the average values of the survey forecasts for the indicated forecast horizon along with the corresponding average of the market-implied expected inflation for the same horizon. Inflation forecasts are expressed as percentages. The sample period is July 2004 to July 2014.

<table>
<thead>
<tr>
<th>Forecast Horizon</th>
<th>Survey</th>
<th>Frequency</th>
<th>Survey Forecast</th>
<th>Market-Implied Forecast</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Year</td>
<td>SPF</td>
<td>Quarterly</td>
<td>2.12</td>
<td>1.78</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>Livingston</td>
<td>Semiannual</td>
<td>2.13</td>
<td>1.60</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Michigan</td>
<td>Monthly</td>
<td>3.23</td>
<td>1.80</td>
<td>120</td>
</tr>
<tr>
<td>5 Years</td>
<td>Michigan</td>
<td>Monthly</td>
<td>2.91</td>
<td>2.31</td>
<td>106</td>
</tr>
<tr>
<td>10 Years</td>
<td>SPF</td>
<td>Quarterly</td>
<td>2.40</td>
<td>2.60</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>Livingston</td>
<td>Semiannual</td>
<td>2.42</td>
<td>2.56</td>
<td>20</td>
</tr>
</tbody>
</table>
### Table 7

*Summary Statistics for Inflation Volatility.* This table reports summary statistics for the volatility of the annualized inflation rate for the indicated horizons. Horizon is expressed in years. Inflation rates are expressed as percentages. The sample consists of daily observations for the period from July 23, 2004 to July 29, 2014.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.385</td>
<td>0.403</td>
<td>0.818</td>
<td>1.373</td>
<td>2.973</td>
<td>1210</td>
</tr>
<tr>
<td>2</td>
<td>1.400</td>
<td>0.422</td>
<td>0.715</td>
<td>1.386</td>
<td>2.365</td>
<td>1103</td>
</tr>
<tr>
<td>3</td>
<td>1.342</td>
<td>0.356</td>
<td>0.685</td>
<td>1.305</td>
<td>2.231</td>
<td>1222</td>
</tr>
<tr>
<td>5</td>
<td>1.351</td>
<td>0.266</td>
<td>0.688</td>
<td>1.399</td>
<td>2.186</td>
<td>1075</td>
</tr>
<tr>
<td>7</td>
<td>1.396</td>
<td>0.237</td>
<td>0.775</td>
<td>1.421</td>
<td>2.073</td>
<td>1124</td>
</tr>
<tr>
<td>10</td>
<td>1.455</td>
<td>0.232</td>
<td>0.932</td>
<td>1.432</td>
<td>2.032</td>
<td>1214</td>
</tr>
<tr>
<td>12</td>
<td>1.445</td>
<td>0.197</td>
<td>1.019</td>
<td>1.448</td>
<td>1.910</td>
<td>1218</td>
</tr>
<tr>
<td>15</td>
<td>1.420</td>
<td>0.201</td>
<td>1.072</td>
<td>1.465</td>
<td>1.874</td>
<td>1198</td>
</tr>
<tr>
<td>20</td>
<td>1.420</td>
<td>0.206</td>
<td>0.991</td>
<td>1.474</td>
<td>1.842</td>
<td>1121</td>
</tr>
<tr>
<td>30</td>
<td>1.397</td>
<td>0.134</td>
<td>0.796</td>
<td>1.414</td>
<td>1.708</td>
<td>789</td>
</tr>
</tbody>
</table>
Table 8

Summary Statistics for Deflation Probabilities. This table reports summary statistics for the probability of the average inflation rate being below zero for the indicated horizons. Horizon is expressed in years. Probabilities are expressed as percentages. The sample consists of daily observations for the period from October 5, 2009 to July 29, 2014.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.73</td>
<td>10.66</td>
<td>0.14</td>
<td>10.61</td>
<td>41.73</td>
<td>1210</td>
</tr>
<tr>
<td>2</td>
<td>11.36</td>
<td>9.06</td>
<td>0.16</td>
<td>8.44</td>
<td>34.00</td>
<td>1103</td>
</tr>
<tr>
<td>3</td>
<td>8.32</td>
<td>7.01</td>
<td>0.06</td>
<td>5.93</td>
<td>27.70</td>
<td>1222</td>
</tr>
<tr>
<td>5</td>
<td>6.34</td>
<td>4.36</td>
<td>0.04</td>
<td>5.81</td>
<td>22.23</td>
<td>1075</td>
</tr>
<tr>
<td>7</td>
<td>5.52</td>
<td>3.18</td>
<td>0.10</td>
<td>5.27</td>
<td>18.27</td>
<td>1124</td>
</tr>
<tr>
<td>10</td>
<td>5.23</td>
<td>2.82</td>
<td>0.31</td>
<td>4.51</td>
<td>11.93</td>
<td>1214</td>
</tr>
<tr>
<td>12</td>
<td>4.83</td>
<td>2.41</td>
<td>0.66</td>
<td>4.34</td>
<td>11.80</td>
<td>1218</td>
</tr>
<tr>
<td>15</td>
<td>4.27</td>
<td>2.37</td>
<td>0.48</td>
<td>3.94</td>
<td>11.77</td>
<td>1198</td>
</tr>
<tr>
<td>20</td>
<td>3.96</td>
<td>2.30</td>
<td>0.25</td>
<td>3.83</td>
<td>12.94</td>
<td>1121</td>
</tr>
<tr>
<td>30</td>
<td>2.77</td>
<td>1.71</td>
<td>0.00</td>
<td>2.34</td>
<td>9.60</td>
<td>789</td>
</tr>
</tbody>
</table>
### Table 9

**Summary Statistics for the Pricing of Deflation Tail Risk.** This table reports the means of the ratio of the probability of inflation being below the indicated threshold under the pricing measure divided by the probability of the same event under the actual measure. The mean is taken over only the observations where the probability of the event is greater than 0.01 percent under the actual measure. Horizon is expressed in years. The sample consists of daily observations for the period from October 5, 2009 to July 29, 2014.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Probability Inflation $&lt; 0.00$</th>
<th>Probability Inflation $&lt; -1.00$</th>
<th>Probability Inflation $&lt; -2.00$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>$N$</td>
<td>Mean</td>
</tr>
<tr>
<td>1</td>
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<td>1210</td>
<td>1.303</td>
</tr>
<tr>
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<td>1.287</td>
<td>1103</td>
<td>1.518</td>
</tr>
<tr>
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<td>1.373</td>
<td>1222</td>
<td>1.704</td>
</tr>
<tr>
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<td>1075</td>
<td>1.682</td>
</tr>
<tr>
<td>7</td>
<td>1.251</td>
<td>1124</td>
<td>1.495</td>
</tr>
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<td>10</td>
<td>1.139</td>
<td>1214</td>
<td>1.287</td>
</tr>
<tr>
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<td>788</td>
<td>2.052</td>
</tr>
</tbody>
</table>
Table 10

Results from the Regression of Monthly Changes in Deflation Probabilities on Financial and Macroeconomic Variables. This table reports the $t$-statistics and adjusted $R^2$s from the regression of monthly changes in the deflation probabilities for the indicated horizon on the monthly changes in the following variables: the spread between three-month Libor and the overnight index swap (OIS) rate, the five-year swap spread, the VIX volatility index, the CDX North American Investment Grade CDS Index, the return on the CRSP value-weighted stock index, the five-year U.S. Treasury CDS spread, the five-year German CDS spread, industrial production (IP, percentage change), the unemployment rate (Unemp), and the Conference Board’s Consumer Confidence Index (Conf). Horizon is measured in years. The $t$-statistics are based on the Newey-West estimator of the covariance matrix (three lags). The superscript $\ast\ast$ denotes significance at the five-percent level; the superscript $\ast$ denotes significance at the ten-percent level. The sample consists of monthly observations for the period from October 2009 to July 2014.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Libor Spread</th>
<th>Swap Spread</th>
<th>VIX</th>
<th>CDX Return</th>
<th>Trsy CDS</th>
<th>German CDS</th>
<th>IP</th>
<th>Unemp</th>
<th>Conf</th>
<th>Adj. $R^2$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.55</td>
<td>2.04$\ast\ast$</td>
<td>-1.35</td>
<td>0.05</td>
<td>-2.40$\ast\ast$</td>
<td>-0.96</td>
<td>0.50</td>
<td>1.05</td>
<td>0.24</td>
<td>-2.34$\ast\ast$</td>
<td>0.251</td>
</tr>
<tr>
<td>2</td>
<td>2.29$\ast\ast$</td>
<td>2.11$\ast\ast$</td>
<td>-2.43$\ast\ast$</td>
<td>-0.80</td>
<td>-3.85$\ast\ast$</td>
<td>-1.84$\ast$</td>
<td>1.81$\ast$</td>
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<td>-0.52</td>
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</tr>
<tr>
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<td>-5.60$\ast\ast$</td>
<td>-1.72$\ast$</td>
<td>2.32$\ast\ast$</td>
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<td>-0.79</td>
<td>-0.51</td>
<td>0.455</td>
</tr>
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<td>0.22</td>
<td>-2.29$\ast\ast$</td>
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<td>-2.69$\ast\ast$</td>
<td>0.29</td>
<td>1.52</td>
<td>0.72</td>
<td>-2.27$\ast\ast$</td>
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<td>0.419</td>
</tr>
<tr>
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<td>0.55</td>
<td>-1.51</td>
<td>1.14</td>
<td>-1.15</td>
<td>-0.29</td>
<td>2.19$\ast\ast$</td>
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<td>-1.94</td>
<td>-0.13</td>
<td>0.277</td>
</tr>
<tr>
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<td>0.82</td>
<td>-1.78$\ast$</td>
<td>-0.31</td>
<td>-0.82</td>
<td>-0.56</td>
<td>1.15</td>
<td>1.64</td>
<td>-0.08</td>
<td>-0.003</td>
</tr>
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<td>-0.39</td>
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<td>1.96$\ast$</td>
<td>0.84</td>
<td>0.95</td>
<td>1.83$\ast$</td>
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<td>-1.99$\ast$</td>
<td>0.45</td>
<td>0.155</td>
</tr>
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<td>0.84</td>
<td>0.24</td>
<td>1.56</td>
<td>1.19</td>
<td>0.79</td>
<td>1.42</td>
<td>0.18</td>
<td>-1.31</td>
<td>-0.03</td>
<td>0.053</td>
</tr>
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<td>20</td>
<td>0.59</td>
<td>1.35</td>
<td>-1.70$\ast$</td>
<td>1.39</td>
<td>-0.04</td>
<td>-0.47</td>
<td>0.62</td>
<td>1.68$\ast$</td>
<td>-1.54</td>
<td>-2.12$\ast\ast$</td>
<td>0.204</td>
</tr>
<tr>
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<td>-0.11</td>
<td>0.31</td>
<td>2.30$\ast\ast$</td>
<td>1.53</td>
<td>-0.41</td>
<td>-0.40</td>
<td>2.33$\ast\ast$</td>
<td>-2.08$\ast\ast$</td>
<td>-2.74$\ast\ast$</td>
<td>0.492</td>
</tr>
</tbody>
</table>
Table 11

Summary Statistics for the Probabilities of Inflationary Scenarios. This table reports summary statistics for the probability of the average inflation rate being above the indicated thresholds for the respective horizons. Horizon is expressed in years. Probabilities are expressed as percentages. The sample consists of daily observations for the period from October 5, 2009 to July 29, 2014.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Probability Inflation &gt; 4.00</th>
<th>Probability Inflation &gt; 5.00</th>
<th>Probability Inflation &gt; 6.00</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Min.</td>
<td>Max</td>
<td>Mean</td>
</tr>
<tr>
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<td>5.14</td>
<td>0.23</td>
<td>22.20</td>
<td>1.40</td>
</tr>
<tr>
<td>2</td>
<td>11.36</td>
<td>0.16</td>
<td>34.00</td>
<td>6.33</td>
</tr>
<tr>
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<td>7.09</td>
<td>0.59</td>
<td>19.56</td>
<td>1.90</td>
</tr>
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<td>0.89</td>
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<td>11.91</td>
<td>2.36</td>
<td>23.13</td>
<td>3.48</td>
</tr>
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<td>25.67</td>
<td>4.66</td>
</tr>
<tr>
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<td>15.03</td>
<td>5.50</td>
<td>24.34</td>
<td>4.72</td>
</tr>
<tr>
<td>15</td>
<td>15.53</td>
<td>5.09</td>
<td>24.04</td>
<td>4.82</td>
</tr>
<tr>
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Table 12

Summary Statistics for the Pricing of Inflation Tail Risk. This table reports the means of the ratio of the probability of inflation being above the indicated threshold under the pricing measure divided by the probability of the same event under the actual measure. The mean is taken over only the observations where the probability of the event is greater than 0.01 percent under the actual measure. Horizon is expressed in years. The sample consists of daily observations for the period from October 5, 2009 to July 29, 2014.

<table>
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<th>Horizon</th>
<th>Probability Inflation &gt; 4.00</th>
<th>Probability Inflation &gt; 5.00</th>
<th>Probability Inflation &gt; 6.00</th>
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<td>N</td>
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<td>1.099</td>
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