A theory of initiation of takeover contests*

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Abstract

We study strategic initiation of takeover contests by potential bidders and the target. After discovering the target, a bidder can approach it thereby putting it “in play.” The target can preempt bidders and put itself for sale. We argue that the decision to approach the target reveals some information about the initiating bidder’s valuation of the target. In “common-value” takeover contests, such as battles between two financial bidders, this disincentivizes the initiating bidder from approaching the target. In pure common-value contests, unraveling occurs: Each bidder never approaches the target, no matter how high his valuation is. By contrast, this effect is limited in “private-value” takeover contests, such as battles between two strategic bidders. Our analysis has implications for how bidders select targets, which deals are bidder- or target-initiated, and how bidding strategies depend on whether a bidder initiated the contest or was solicited by the target.

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1 Introduction

Acquisitions and intercorporate asset sales are some of the most important decisions that management of corporations faces. Competition to acquire a company or an asset often resembles an auction, either formal or informal. As a consequence, many researchers have applied the tools of auction theory to study various aspects of the corporate control market, such as the split of acquisition gains between bidders and targets, preemptive bids, toeholds, as well as optimal mechanisms to sell a company.\(^1\) To focus on the insights about the auction stage, the literature usually examines a situation when the company or the asset is already up for sale.

While exogeneity of an auction taking place may be an innocuous assumption in situations like Treasury auctions that occur with a known frequency, it is unlikely to apply to the market for corporate control. In practice, a takeover contest can be either bidder-initiated, when a potential bidder approaches the target’s board which then decides to auction the company off, or target-initiated, when the target’s board decides to auction the company off without being approached by a potential bidder first. To give a flavor of this heterogeneity, consider the following two large recent deals. The acquisition of Taleo, a provider of cloud-based talent management solutions, by Oracle on February 9, 2012 for $1.9 billion presents an example of a bidder-initiated takeover auction. In January 2011, a CEO of a publicly traded technology company, referred in a deal background as Party A, contacted Taleo expressing an interest in acquiring it. Following this contact, Taleo hired a financial adviser that conducted an auction, engaging four more bidders. Oracle was the winning bidder in the auction, ending up acquiring Taleo. By contrast, the acquisition of Blue Coat Systems, a provider of Web security, by a private equity firm Thoma Bravo on December 9, 2011 for $1.1 billion presents an example of a target-initiated takeover auction. In early 2011, Elliot Associates, an activist hedge fund, amassed 9% ownership stake in Blue Coat and forced its board to auction the company. Twelve bidders participated in the

auction, and Thoma Bravo was the winner. Overall, not only there exists a considerable heterogeneity by initiator of the contest, but also it appears to be far from random. For example, in the sample of Fidrmuc et al. (2012), acquisitions by strategic acquirers are more likely to be bidder-initiated, while acquisitions by private equity firms are more likely to be target-initiated.

In this paper, we provide a simple theory of how bidders and targets choose to initiate takeover auctions. We argue that the decision of a potential bidder to approach the target reveals information about his valuation to the target and other potential bidders, which has an important impact on the outcome of the auction, thereby affecting the decision of the potential bidder to approach the target in the first place. We show that the importance of informational effects of a bidder’s decision to approach the target heavily depends on the the sources of bidders’ valuations. In “common-value” battles, e.g., when two financial bidders compete with each other to acquire the target, the information revealed through a bidder’s decision to approach the target has a large negative effect on his expected acquisition gains, which disincentivizes both bidders from initiating the contest. This result contrasts with “private-value” battles, e.g., a competition between two strategic bidders, where, as we show, this informational effect is limited. Our analysis has implications for the dynamics of the corporate control market, both on aggregate and individual firm level. In particular, we derive conditions under which deals can be bidder- or target-initiated, or not initiated at all. We also characterize the equilibrium bids and the division of surplus among the target, the initiating bidder, and the non-initiating bidders.

We start with the simplest setting in which there is room for strategic initiation decisions by bidders only. In addition to one target, there are two potential bidders, who are initially unaware of the target. At the initial date, one of the bidders can discover the target with some probability. Once a bidder discovers the target, he can approach it thereby putting it “in play”. At this point, the target invites the other bidder to participate in the competitive process. The other bidder then also obtains a private signal, and the two bidders compete in a sealed-bid first-price auction. We initially assume that a takeover contest must be
bidder-initiated and then allow the target to preempt bidders and put itself up for sale without being approached by a bidder. We consider two valuation environments. In the common-value case, both bidders share the same actual value of the target, but may have different estimates of this value because they get different private signals. In the private-value case, the private signal of a bidder affects only his own valuation of the target, but not his rival’s valuation. To achieve tractability and provide intuition, we first consider a static model, in which the bidder that discovers the target faces a single chance to approach it. Later, we extend the model to a fully dynamic setup, albeit only with binary signals.

Our central result is that bidder-initiated takeovers unravel if bidders have common values. In the unique equilibrium no bidder ever initiates a takeover auction when he discovers the target, no matter how high his estimate of the value created in the deal is. This result occurs because in the common-value framework the lowest type of the bidder that initiates a takeover contest must obtain zero expected surplus from the auction. Intuitively, if the bidder approaches the target if and only if his signal is above some $\hat{s}$, then the other bidder re-evaluates his estimate of the target value upwards when he observes that the auction is bidder-initiated. In the case of pure common values, it means that type $\hat{s}$ of the initiating bidder only wins the auction, when the type of the rival bidder is the lowest, and in this case he pays the whole value of the target, obtaining zero surplus. Because the argument holds for any $\hat{s}$, the only equilibrium is the one in which bidder-initiated takeovers never happen. This implies that in common-value cases, such as contests among financial bidders, takeover contests tend to be target-initiated. Only if the common-value bidder is able to obtain a toehold in the target at a low price, he may find it optimal to initiate the takeover contest himself.

In contrast, in the case of private values, although the decision of a bidder to approach the target reveals his information to his rival, the rival does not update his valuation of the target upon learning that the auction is bidder-initiated. With some probability, the valuation of the rival bidder is low enough, so that even type $\hat{s}$ of the initiating bidder is

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2This is more likely to happen for targets which are not expected to be acquired, e.g., in industries with many similar firms.
able to acquire the target for cheap, implying positive expected surplus from the auction. The boundary type \( \hat{s} \) is thus defined as the type of a bidder whose expected surplus from the auction equals the cost of initiating the takeover auction. Note that even though the rival bidder does not update his valuation when observing that the auction is bidder-initiated, he bids more aggressively, because he knows he faces a stronger competitor. This reduces the surplus of the initiating bidder somewhat relative to the case of the target-initiated auction, but does not lead to unraveling, as in the case of common values.

We proceed by analyzing what happens if the target can preempt bidders and initiate the auction process itself. In practice, while such initiation gives the target flexibility, it also entails costs, either direct costs of hiring the investment bank to search for potential bidders or indirect costs of sending a signal that the target is desperate. Because bidders never initiate contests in the common-values case, paying the cost and initiating the contest itself is the only option available to the target if it wants to sell itself. Thus, deals will either be target-initiated or never take place. Because initiation by the target makes deals insensitive to information of bidders, many high-value deals never happen in equilibrium: bidders have no incentives to act based on their high signals, while the target does not know that bidders’ signals are high. This result provides a novel and intuitive explanation of why there are many completed target-initiated contests in practice despite the traditional view that the bidders, if they are not ready to initiate by themselves, can always walk away from the deal. In the presence of the common value component in their valuation for the target, the bidders may not be willing to initiate contests by themselves but are ready to enter target-initiated contests which alleviate the adverse selection problem.

In the private-value case, the equilibrium is more interesting. Because of the possibility of being approached by a bidder, the target has the option of not announcing the auction immediately. This leads to the possibility of both bidder- and target-initiated takeovers. In the dynamic extension analyzed in the paper, the equilibrium often takes a threshold form: the target waits for some time and, if it has not been approached, initiates the takeover contest itself. Taken together, our theoretical results are consistent with empirical evidence.
on target- and bidder-initiated strategic and private-equity deals, presented in Fidrmuc et.
al (2012): approximately 60% (35%) of strategic (private-equity) deals are initiated by the
bidders. Our explanation of this large discrepancy is that financial but not strategic bidders
have an important common value component in their valuations for targets.

Our paper is related to the vast literature that studies takeover contests as auctions.
Takeovers are modeled using both private-value framework (e.g., Fishman (1988), Burkart
(1995), and Singh (1998)) and common-value framework (e.g., Bulow, Huang, and Klem-
perer (1999), and Povel and Singh (2006)). In our interpretation of the private-value frame-
work as a competition between strategic bidders and the common-value framework as a
competition between financial bidders, we follow Bulow, Huang, and Klemperer (1999).
To our knowledge, none of these papers studies endogenous initiation of takeover contests.
This dynamic feature makes our paper also related to the recent literature on the timing of
mergers and acquisitions.\(^3\) It is, however, difficult to relate these papers to the literature on
auctions, as they assume that the bidder and the target share the same information. The
search and discovery feature of our model relates to Rhodes-Kropf and Robinson (2008)
who develop a search model of the M&A market. Finally, because the decision of a bidder
to approach the target makes the two bidders asymmetric in their information sets, our
paper is related to the literature on exogenous auctions with asymmetrically-informed bid-
ders. The most closely related papers in this literature are Campbell and Levin (2000) and
Kim (2008) that study first-price common-value auctions in which bidders are differentially
informed. Our contribution to this literature is that bidder asymmetries arise endogenously
through decision of the initiating bidder to approach the target.

The remainder of the paper is organized as follows. Section 2 describes the setup of
the model. Section 3 assumes that takeovers can only be bidder-initiated and solves for
the equilibrium. Section 4 allows for both bidder- and target-initiated takeovers. Section
5 describes implications of the model. Section 6 considers a dynamic extension. Section 7
discusses other elements missing from the main model. Finally, Section 8 concludes.

\(^3\) Lambrecht (2004), Lambrecht and Myers (2007), Morellec and Zhdanov (2005), Hackbarth and Morellec
(2008), Hackbarth and Miao (2012), and Gorbenko and Malenko (2013).
2 The Model

There are three players: a risk-neutral seller and two potential risk-neutral bidders, \( i \in \{1, 2\} \). For convenience, we refer to the seller as the “target”, having in mind the market for mergers and acquisitions, but our analysis also applies to transactions of other assets. The value of the target as a stand-alone company is normalized to zero.

While the target does exist, the bidders are initially unaware of the target, and need to discover it. The timeline is as follows. At date 0, the target is either discovered by a single bidder (probability \( 2p \))\(^4\) or is not discovered (probability \( 1 - 2p \)). The target is equally likely to be discovered by bidder 1 and 2: each event occurs with probability \( p \). The discovery process is aimed at capturing the search activity of acquirers, such as private equity firms.

\(^4\)We assume that both bidders cannot simultaneously discover the target for simplicity. Section 6 presents a dynamic model in which this assumption is rationalized.
and repeated corporate acquirers.

If and when bidder $i$ discovers the target, he observes a private signal $s_i$. Bidders’ signals are independent draws from the same distribution. Without loss of generality, we normalize so that both $s_1$ and $s_2$ are uniformly distributed over $[0, 1]$.

Conditional on both signals, the value of the target to bidder $i$ is $v(s_i, s_{-i})$, where $s_{-i}$ is the signal of his rival.

**Assumption 1.** Function $v(s_i, s_{-i})$ is continuous in both variables, strictly increasing in $s_i$, and satisfies $v(s_i, s_{-i}) > 0$ for all $(s_i, s_{-i}) \in [0, 1]^2$.

Assumption 1 is a standard assumption in auction theory. Continuity means that there are no “gaps” in possible valuations of the target. Strict monotonicity in the first variable means that a higher private signal is always good news about the bidder’s valuation. Finally, $v(s_i, s_{-i}) > 0$ for any combination of $s_i$ and $s_{-i}$ means that an acquisition is always the efficient outcome, which is convenient for exposition but not necessary for our results. This valuation structure follows the general symmetric model of Milgrom and Weber (1982).

This setup embeds two valuation structures commonly used in the literature:

- **The private-values framework.** This is the case if and only if $v(s_i, s_{-i}) = v(s_i)$. The unique feature of private values is that a bidder’s signal provides information only about his own valuation, but not about the valuation of his competitor.

- **The common-values framework.** This is the case if and only if $v(s_i, s_{-i}) = v(s_{-i}, s_i)$. Conditional on observing both signals, both bidders have the same valuation of the target. However, bidders differ in their assessment of the value of the target, because their private signals are different.

For the most of the paper, we focus on these two valuation structures. There are two compelling interpretations of common values versus private values that we employ.

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5This normalization is without loss of generality, because we do not assume a specific functional form for the mapping of signals into valuations.
Our first interpretation relates to different types of bidders. Following Bulow, Huang, and Klemperer (1999), we can interpret the common-value (private-value) auction as a takeover battle between two financial (strategic) bidders. Intuitively, financial bidders tend to use the same strategies (i.e., have “common” values) after they acquire the target, but may have different estimates of potential gains (i.e., have different signals). In contrast, because synergies that one strategic bidder expects to achieve from acquiring the target are often bidder-specific, they provide little information about valuation of the target to the other bidder. This interpretation is somewhat consistent with the finding of Gorbenko and Malenko (2012) that conditional on observed characteristics of the target, valuations of strategic bidders are more dispersed than valuations of financial bidders.

Our second interpretation relates to different types of targets rather than bidders. On a very high level, value can be created in acquisitions either because the incumbent management of the target is inefficient or because the target and the acquirer have synergies that cannot be realized by the acquirer as a stand-alone. Acquisitions of the first type are common-value deals, while acquisitions of the second type are private-value deals.

At date 1, if bidder $i$ discovers the target, he can approach the target thereby putting it “in play.” To approach the target, bidder $i$ needs to incur a small positive cost $c$. It captures both direct costs, such as time spent and fees paid to lawyers for drafting an appropriate letter of intent, as well as the opportunity costs of not making other acquisitions while the process is taking place. Our key result about unraveling in common-value auctions does not depend on the magnitude of $c$, as long as it is positive. Economically, the assumption we require is that the bidder does not initiate the takeover contest if he expects to get zero surplus from the auction.\(^6\)

If the target is put in play and bidder $j$ has not discovered it earlier, he immediately learns about the target’s existence and observes a private signal $s_j$. Once the contest is initiated, the target is sold through a sealed-bid first-price auction. Each of the two bidders simultaneously submits his bid to the target in a concealed fashion. The two bids are

\(^6\)Without this assumption, there will multiple equilibria in the model, as bidders expecting to obtain zero surplus may initiate contests in equilibrium, which will also change the strategy of other bidders.
compared, and the bidder that submitted a higher bid acquires the target and pays the bid he submitted. All shareholders of the target are assumed to be willing to sell their shares to the highest bidder, as long as her bid exceeds zero, which always happens in equilibrium. In particular, we ignore the free-rider problem of Grossman and Hart (1980).

Three versions of the model are considered in the following sections. We start with the setup described above. Then, we extend it by allowing the target to put itself for sale without being approached by a bidder first. Finally, we consider a fully dynamic setup in which both bidders and the target make continuous decisions about the initiation.

3 Bidder-Initiated Takeovers

First, we consider the problem in which all takeovers can only be initiated by bidders. Suppose that the target was discovered by a bidder. Without loss of generality, we refer to this bidder as bidder 1. We first consider the common-value model and show that the informational effect of initiation puts the initiating bidder in an adverse position. In fact, this effect is so extreme that in the unique equilibrium bidders never approach the target, and the contest is never initiated. As we show next, this result contrasts sharply with the private-value model, in which the informational effect of initiation has no effect on the initiating bidder’s expected payoff from the auction.

3.1 The Common-Value Case

Consider the case of pure common values, \( v(s_1, s_2) = v(s_2, s_1) \). Note that the payoff of the initiating bidder is increasing in his signal. Therefore, in any equilibrium the initiation strategy must follow the threshold rule: if type \( s \) initiates, then any type above \( s \) must also initiate. Let \( \hat{s} \in [0, 1] \) denote the lowest type of the bidder that initiates the auction after discovering the target. Then, conditional on the auction being initiated, the non-initiating bidder believes that \( s_1 \) is drawn from uniform distribution over \([\hat{s}, 1]\). By contrast, bidder 1 believes that \( s_2 \) is drawn from uniform distribution over \([0, 1]\). Thus, even though both
bidders are ex-ante symmetric, initiation endogenously creates an asymmetry in that bidder 1 is, on average, a higher type than bidder 2.

Next, we solve for the equilibrium at the auction stage. Conjecture that the equilibrium in pure strategies exists. Let \( \beta_1 (s_1, \hat{s}) \) denote the bid of the initiating bidder 1 with signal \( s_1 \geq \hat{s} \), given that types \( \hat{s} \) and above initiate the contest. Similarly, let \( \beta_2 (s_2, \hat{s}) \) denote the bid of the non-initiating bidder 2 with signal \( s_2 \), given that bidder 1 initiates the auction if and only if \( s_1 \geq \hat{s} \). Let \( b = \beta_1 (1, \hat{s}) = \beta_2 (1, \hat{s}) \) be the common highest bid submitted by both bidders.\(^7\) The expected payoff of each bidder \( i \) when her signal is \( s_i \) and bid is \( b \) is

\[
\Pi_i (b, s_i, \hat{s}) = \int_0^{\phi_i(b, \hat{s})} (v(s_i, x) - b) \, dx,
\]

\( i = 1, 2 \)

(1)

(2)

where \( \phi_i \equiv \beta_i^{-1} \) is the inverse in \( s_i \) of bidder \( i \)'s bidding function. Intuitively, the expected payoff of a bidder from the auction equals his valuation, which depends on the realization of the competitor’s signal, less his bid, integrated over all realizations of the competitor’s signal, such that the bidder is the winner.

Taking the first-order conditions of (1) and (2), we obtain

\[
(v(s_1, \phi_2(b, \hat{s})) - b) \frac{\partial \phi_2(b, \hat{s})}{\partial b} - \phi_2(b, \hat{s}) = 0,
\]

(3)

\[
(v(\phi_1(b, \hat{s}), s_2) - b) \frac{\partial \phi_1(b, \hat{s})}{\partial b} - \phi_1(b, \hat{s}) + \hat{s} = 0.
\]

(4)

In equilibrium, \( b = \beta_1 (s_1, \hat{s}) \) must satisfy (3) and \( b = \beta_2 (s_2, \hat{s}) \) must satisfy (4), which imply \( s_i = \phi_i(b, \hat{s}) \). Plugging in and rearranging the terms, we obtain that for all \( b < \tilde{b} \) the

\(^7\)The proof that \( \beta_1 (1, \hat{s}) = \beta_2 (1, \hat{s}) \) is straightforward. Suppose \( \beta_1 (1, \hat{s}) > \beta_2 (1, \hat{s}) \). Then, types of bidder 1 close enough to 1 can reduce their bids and still win the auction with probability 1. Thus, \( \beta_1 (1, \hat{s}) > \beta_2 (1, \hat{s}) \) cannot occur in equilibrium. Similarly, \( \beta_1 (1, \hat{s}) < \beta_2 (1, \hat{s}) \) cannot occur in equilibrium.
following differential equations must hold:

\[
\frac{\partial \phi_2 (b, \hat{s})}{\partial b} = \frac{\phi_2 (b, \hat{s})}{v (\phi_1 (b, \hat{s}), \phi_2 (b, \hat{s})) - b'}, \tag{5}
\]

\[
\frac{\partial \phi_1 (b, \hat{s})}{\partial b} = \frac{\phi_1 (b, \hat{s}) - \hat{s}}{v (\phi_1 (b, \hat{s}), \phi_2 (b, \hat{s})) - b'}. \tag{6}
\]

The intuition behind (5)–(6) is as follows. Bidder \(i\) faces a trade-off. On one hand, by slightly increasing his bid above \(b\), he increases the likelihood of winning by \(\partial \phi_{-i} (b, \hat{s}) / \partial b\). In this small-probability event, he will marginally outbid the other bidder, which implies that bidder \(i\) will obtain the surplus of \(v (\phi_1 (b, \hat{s}), \phi_2 (b, \hat{s})) - b\). On the other hand, by slightly increasing his bid above \(b\), bidder \(i\) will pay more whenever he wins the auction. Thus, the marginal cost of increasing the bid is the probability of winning, which is \(\phi_2 (b, \hat{s})\) for bidder 1 and \(\phi_2 (b, \hat{s}) - \hat{s}\) for bidder 2. In equilibrium, the mapping between bids and signals is such that the marginal costs and benefits are equal, yielding (5)–(6).

The system of equations (5)–(6) is solved subject to the boundary conditions, which are to be determined. Let \([b, \bar{b}]\) denote the interval of possible bids. It must be the same for both bidders, as otherwise one of the bidders finds it optimal to deviate. The upper boundary implies \(1 = \phi_1 (\bar{b}, \hat{s}) = \phi_2 (\bar{b}, \hat{s})\). Consider the lower boundary \(\underline{b}\). First, it cannot be below \(v (\hat{s}, 0)\), because either bidder 1 of type \(\hat{s}\) or bidder 2 of type 0 would find it optimal to deviate and submit a marginally higher bid. By doing this, either such bidder can increase the probability of winning from zero to a positive number, and thus obtain a positive expected surplus instead of zero. Therefore, \(\underline{b} \geq v (\hat{s}, 0)\). Second, \(\underline{b}\) cannot be above \(v (\hat{s}, 0)\), because such \(\underline{b}\) would imply that low enough types of bidder 2, such that \(v (\hat{s}, s_2) < \underline{b}\), obtain a negative surplus in equilibrium. Thus, we obtain the following lemma:

Lemma 1 (equilibrium in the bidder-initiated CV auction). Necessary and sufficient conditions for bidding strategies \(\beta_1 (s_1, \hat{s})\) and \(\beta_2 (s_2, \hat{s})\) to constitute an equilibrium
are that $\beta_1(s_1, \hat{s})$ and $\beta_2(s_2, \hat{s})$ are increasing functions that satisfy

$$
\frac{\partial \beta_1(s_1, \hat{s})}{\partial s_1} = \frac{v(s_1, \phi_2(\beta_1(s_1, \hat{s}), \hat{s})) - \beta_1(s_1, \hat{s})}{s_1 - \hat{s}},
$$

(7)

$$
\frac{\partial \beta_2(s_2, \hat{s})}{\partial s_2} = \frac{v(\phi_1(\beta_2(s_2, \hat{s}), \hat{s}), s_2) - \beta_2(s_2, \hat{s})}{s_2},
$$

(8)

with boundary conditions given by

$$
\beta_1(\hat{s}, \hat{s}) = \beta_2(0, \hat{s}) = v(\hat{s}, 0),
$$

(9)

and

$$
\beta_1(1, \hat{s}) = \beta_2(1, \hat{s}).
$$

(10)

The equilibrium in the auction implies the payoff of bidder 1 with type $\hat{s}$ in equilibrium is $\Pi_1(\beta_1(\hat{s}, \hat{s}), \hat{s}, \hat{s}) = 0$. The intuition is as follows. Bidder 2 knows that bidder 1 approaches the target if and only if his signal is at least $\hat{s}$. Therefore, when he sees that the auction was bidder-initiated, he re-evaluates the target to at least $v(\hat{s}, s_2)$, where $s_2$ is his own signal. Similarly, bidder 1 estimates that the target is worth at least $v(s_1, 0)$. Because under no circumstance the target is valued less than $v(\hat{s}, 0)$, no bidder in equilibrium bids less than this. However, adverse selection implies that bidder 1 with type $\hat{s}$ wins the auction only when the type of bidder 2 is 0. In this situation, the value of the target is exactly $v(s_1, 0)$, leaving bidder 1 without surplus. Because bidder 1 with type $\hat{s}$ obtains zero surplus, he never wants to incur a cost to approach the target. Because the argument holds for any $\hat{s}$, there exists no equilibrium in which a bidder initiates the auction. Proposition 1 summarizes the above intuition:

**Proposition 1.** In the unique Bayes-Nash equilibrium, a bidder never approaches the target after she discovers it. The off-equilibrium beliefs are such that if a bidder approaches the target, her type is 1.
It is instructive to consider an example of the additive valuation as the most commonly used common-value specification.

**Example:** \( v(s_1, s_2) = \frac{1}{2} (s_1 + s_2) \). Then, the equilibrium bidding strategies are linear:

\[
\beta_1(s_1, \hat{s}) = -\frac{\hat{s}^2}{4(1-\hat{s})} + \frac{2-\hat{s}}{4(1-\hat{s})} s,
\]

\[
\beta_2(s, \hat{s}) = \frac{\hat{s}}{2} + \frac{2-\hat{s}}{4}s.
\]

The common range of bids is \( [\frac{\hat{s}}{2}, \frac{\hat{s}+2}{4}] \). The equilibrium bids are plotted on the left panel of Figure 2 for \( \hat{s} = 0.5 \).

![Plot of bid as a function of signal](image1)

![Plot of expected bidder surplus as a function of signal](image2)

**Figure 2:** Equilibrium bids and expected payoffs of bidders in a bidder-initiated common-value auction. The left panel plots the equilibrium bids as functions of signals for the initiating bidder (the blue normal line) and the other bidder (the red dashed line). The right panel plots the corresponding expected surpluses of each bidder.

An important result from the bidding stage is that the initiating bidder with type \( \hat{s} \) obtains a zero expected payoff. As we show in the next section, this result is unique to the common-value setting. The argument behind this result generalizes the logic of Engelbrecht-Wiggans, Milgrom, and Weber (1983) who show that a bidder that has access to only public information always obtains zero surplus in equilibrium. Here, bidder 1 does
have proprietary information, because his decision to approach the target only reveals that his signal is at or above a certain level. Therefore, almost every type of bidder 1 does obtain a positive expected surplus, unlike a bidder with public information in Engelbrecht-Wiggans, Milgrom, and Weber (1983). This can be seen on the right panel of Figure 2. However, the marginal type of bidder 1 that initiates the contest, \( \hat{s} \), obtains zero surplus, because his decision to approach the target makes it common knowledge that his type is at least \( \hat{s} \). Put differently, in the common-value model, bidder 1 obtains expected surplus because of his information rents: To reveal his higher signal through a higher bid, the bidder must be compensated with a higher surplus, which takes the form of a higher probability of winning. However, bidder 1 with type \( \hat{s} \) has no information rent: It is common knowledge that \( s_1 \) is at least \( \hat{s} \), so type \( \hat{s} \) has no lower types to separate from; hence, there is no need to pay him.

In Section 3, we allow the target to initiate the contest without being approached by a bidder first and show that only target-initiated takeovers can happen in the common-value case.

### 3.2 The Private-Value Case

In the private-value model, only a bidder’s own signal impacts his valuation, which we denote as \( v(s_i) \). As in the case of common values, the initiation strategy of the bidder must follow a threshold rule in equilibrium. This is because the expected payoff of a bidder from the auction is increasing in his signal \( s_i \). Let \( \hat{s} \in [0,1] \) denote the lowest type of the bidder that initiates the auction after discovering the target. Then, bidder 2 estimates that \( s_1 \) is drawn from uniform distribution over \( [\hat{s}, 1] \), while bidder 1 estimates that \( s_2 \) is drawn from uniform distribution over \( [0, 1] \).

Consider the auction stage. let \( \gamma_i(s_i, \hat{s}) \) denote the equilibrium bid of bidder \( i \) with signal \( s_i \), assuming that a bidder that discovers the target approaches her if and only if his signal exceeds \( \hat{s} \). The same argument as in the common-values case applies to show that in any equilibrium, it must be that \( \gamma_1(1, \hat{s}) \) is equal to \( \gamma_2(1, \hat{s}) \). Thus, let \( \tilde{g} = \gamma_1(1, \hat{s}) = \gamma_2(1, \hat{s}) \)
denote the common highest bid submitted by both bidders. The expected payoff of each bidder when her signal is \( s_i \) and bid is \( b \) is

\[
\Pi_1(b, s_1, \hat{s}) = f_2(b, \hat{s}) (v(s_1) - b), \tag{11}
\]
\[
\Pi_2(b, s_2, \hat{s}) = \frac{f_1(b, \hat{s}) - \hat{s}}{1 - \hat{s}} (v(s_2) - b), \tag{12}
\]

where \( f_i = \gamma_i^{-1} \) is the inverse in \( s_i \) of bidder \( i \)'s bidding function. Compared to (1)–(2), valuations in equations (11)–(12) do not depend on the realization of the competitor’s signal. As a consequence, the expected payoff of a bidder is his valuation from winning less the bid multiplied by the probability that his bid exceeds that of the competitor. Because bidder 1 is typically stronger than bidder 2, for the same inverse \( f_i \) and bid \( b \), the probability of winning is lower for bidder 2.

Taking the first-order conditions of (11) and (12), respectively, yields

\[
(v(s_1) - b) \frac{\partial f_2(b, \hat{s})}{\partial b} - f_2(b, \hat{s}) = 0, \tag{13}
\]
\[
(v(s_2) - b) \frac{\partial f_1(b, \hat{s})}{\partial b} - f_1(b, \hat{s}) + \hat{s} = 0. \tag{14}
\]

In equilibrium, \( b = \gamma_1(s_1, \hat{s}) \) and \( b = \gamma_2(s_2, \hat{s}) \) must solve (13) and (14), respectively. Therefore, \( s_i = f_i(b, \hat{s}) \). Plugging in and rearranging the terms, we obtain the following system of two differential equations:

\[
\frac{\partial f_2(b, \hat{s})}{\partial b} = \frac{f_2(b, \hat{s})}{v(f_1(b, \hat{s})) - b}, \tag{15}
\]
\[
\frac{\partial f_1(b, \hat{s})}{\partial b} = \frac{f_1(b, \hat{s}) - \hat{s}}{v(f_2(b, \hat{s})) - b}. \tag{16}
\]

The system of differential equations (15)–(16) is similar to (5)–(6) with the difference that valuations in the denominators on the right-hand sides do not depend on both signals of bidders.

The system of equations (15)–(16) is solved subject to the appropriate boundary con-
ditions. Let \([g, \bar{g}]\) be the interval of bids such that any bid \(b > g\) from this interval is potentially a winning bid. The upper boundary implies \(1 = f_1(\bar{g}, \hat{s}) = f_2(\bar{g}, \hat{s})\). The lower boundary implies \(f_1(g, \hat{s}) = \hat{s}\). Finally, optimality of bidder 2 implies \(f_2(g, \hat{s}) = v^{-1}(g)\). These three boundary conditions pin down three constants \(\bar{g}, g, \underline{s}\) and the lowest type of bidder 2 who bids competitively, \(\underline{s}_2 = v^{-1}(g)\). The following lemma summarizes the equilibrium:

**Lemma 2 (equilibrium in the bidder-initiated PV auction).** Necessary and sufficient conditions for bidding strategies \(\gamma_1(s_1, \hat{s})\) and \(\gamma_2(s_2, \hat{s})\) to constitute an equilibrium are that \(\gamma_1(s_1, \hat{s})\) and \(\gamma_2(s_2, \hat{s})\) are increasing functions that satisfy

\[
\frac{\partial \gamma_1(s_1, \hat{s})}{\partial s_1} = \frac{v(f_2(b, \hat{s})) - \gamma_1(s_1, \hat{s})}{s_1 - \hat{s}},
\]

(17)

\[
\frac{\partial \gamma_2(s_2, \hat{s})}{\partial s_2} = \frac{v(f_1(b, \hat{s})) - \gamma_2(s_2, \hat{s})}{s_2},
\]

(18)

with boundary conditions given by

\[
\gamma_1(\hat{s}, \hat{s}) = \gamma_2(\underline{s}_2, \hat{s}) = v(\underline{s}_2),
\]

(19)

and

\[
\gamma_1(1, \hat{s}) = \gamma_2(1, \hat{s}).
\]

(20)

If \(s_2 < \underline{s}_2\), bidder 2 bids any amount below \(v(\underline{s}_2)\).

The equilibrium in the auction implies the payoff of bidder 1 with type \(\hat{s}\) in equilibrium is

\[
\Pi_1(\gamma_1(\hat{s}, \hat{s}), \hat{s}, \hat{s}) = v^{-1}(\gamma_1(\hat{s}, \hat{s}))(v(\hat{s}) - \gamma_1(\hat{s}, \hat{s}))
\]

(21)

\[
= \max_b v^{-1}(b)(v(\hat{s}) - b) > 0.
\]

(22)
By the envelope theorem, the payoff of bidder 1 with the lowest type \( \hat{s} \) is increasing in \( \hat{s} \). The threshold type of bidder 1 that is indifferent between approaching the target and not is such that his equilibrium payoff (21) equals the cost of approaching the target:

\[
v^{-1}(\gamma_1(\hat{s}, \hat{s}))(v(\hat{s}) - \gamma_1(\hat{s}, \hat{s})) = c.
\] (23)

Because the left-hand side of (23) is strictly increasing in \( \hat{s} \) by the envelope theorem and the right-hand side of (23) is a constant, equation (23) uniquely pins down \( \hat{s} \), assuming that \( c \) is not too high. The next proposition summarizes the equilibrium:

**Proposition 2.** In the private-values case, after discovering the target, a bidder approaches it if and only if her signal is above cut-off \( \hat{s} \), given by (23). In the auction stage, the initiating bidder bids \( \gamma_1(s_1, \hat{s}) \), where \( s_1 \geq \hat{s} \) is his signal, while the other bidder bids \( \gamma_2(s_2, \hat{s}) \). \( \gamma_1(s_1, \hat{s}) \) and \( \gamma_2(s_2, \hat{s}) \) are given in Lemma 2.

Proposition 2 establishes that low enough types of bidders do not approach the target also in the private-value case, but the effect is not nearly as extreme as in the common-value case. It is reasonable to assume that the costs of initiation are low relative to potential gains from the acquisition. However, when \( c \) is low, the probability that a bidder does not approach the target is also low. In particular, it converges to zero as \( c \) becomes close to zero. This goes in sharp contrast with the common-value case in which bidders never approach targets, no matter how low the positive costs of initiation are.

As before, it is instructive to illustrate the results using a simple example.

**Example:** \( v(s_i) = s_i \). Then, the marginal type that approaches the target is \( \hat{s} = 2\sqrt{c} \).

Bidding functions are determined from the differential equations

\[
\frac{\partial \gamma_1(s)}{\partial s} = \frac{(\gamma_1(s) - \hat{s}) \gamma_1(s)}{(s - \hat{s})(s - \gamma_1(s))},
\] (24)

\[
\frac{\partial \gamma_2(s)}{\partial s} = \frac{(\gamma_2(s) - \hat{s}) \gamma_2(s)}{s(s - \gamma_2(s))},
\] (25)
with boundary conditions $\gamma_1(\hat{s}) = \hat{s}/2$ and $\gamma_2(\hat{s}/2) = \hat{s}/2$. The common highest bid is $\gamma_1(1) = \gamma_2(1) = (1 - \frac{1}{4} s^2) / (2 - \hat{s})$. Bidder 2 of type below $\hat{s}/2$ bids any amount below $\hat{s}/2$ and loses with probability 1. The equilibrium bids are plotted on Figure 3.

Figure 3: **Equilibrium bids and expected payoffs of bidders in a bidder-initiated private-value auction.** The left panel plots the equilibrium bids as functions of signals for the initiating bidder (the blue normal line) and the other bidder (the red dashed line). The right panel plots the corresponding expected surpluses of each bidder.

## 4 Initiation of Takeovers by Bidders and Targets

So far we have assumed that a takeover can only be bidder-initiated. In practice, many takeover contests are initiated by the target. In these cases, the target and its investment bank approach potentially interested parties inviting them to participate in a process which shares many features with formal auctions. This process is described and modeled by Hansen (2001). We assume that initiating the takeover contest is costly for the target. Otherwise, the solution would be trivial: the target would initiate the takeover contest immediately at the initial date, and bidders would enter the contest having symmetric information. The assumption of costly initiation can be rationalized by both direct costs of hiring the investment bank to contact the bidders, searching for potential bidders and
indirect costs of disclosing public information to competitors of the target or sending a signal to bidders that the target is desperate.

The possibility that a takeover contest can be initiated by the target creates a complicated decision-making problem for bidders and the target. The target may delay initiating the contest herself hoping that it will be approached by bidders. At the same time, a bidder that discovers the target may choose to delay approaching the target hoping that the contest will be initiated by the target or the other bidder. In this section, we consider the simplest possible dynamics. Specifically, assume that before the game analyzed in the previous section starts, the target has an option to pay a fixed cost $I > 0$ and initiate the contest herself. Thus, this setup captures the strategic motive of the target to wait until it is approached by bidders, but not the strategic motive of bidders. A more complete case is analyzed in Section 6, albeit in a setting with only two possible types.

4.1 The Common-Value Case

Proposition 1 implies that for any possible initiation strategy of the target, each bidder has a dominant strategy of never approaching the target. Each bidder only participates in the contest if it is initiated by the target. Therefore, at the initial date the target faces the choice between paying cost $I$ to run an auction and continuing as a stand-alone company.

Suppose that the target initiates the contest. Because the auction is target-initiated, both bidders are symmetric, so we will look for an equilibrium in symmetric bidding strategies. This derivation is standard (e.g., Chapter 6.4 in Krishna, 2010). Let $\beta (s)$ denote the bid of a bidder with signal $s$. The expected payoff of a bidder with signal $s$ who bids $b$ is

$$\Pi (b, s) = \int_0^{\beta^{-1}(b)} (v(s, x) - b) \, dx,$$  (26)

Taking the first-order condition and using the fact that the maximum is reached at $b = \beta (s)$,
we obtain differential equation

\[ \beta'(s) = \frac{v(s,s) - \beta(s)}{s}. \] (27)

This equation is solved subject to the boundary condition \( \beta(0) = v(0,0) \). Intuitively, the lowest type wins the auction only if her competitor also has the lowest signal. The value of the target conditional on this event is \( v(0,0) \). Because under no circumstance the target is valued less than \( v(0,0) \), the bidders compete away the value and bid \( v(0,0) \). Solving (27) subject to \( \beta(0) = v(0,0) \) yields:

**Lemma 3 (equilibrium in the target-initiated CV auction).** If the target initiates the auction, the two bidders are symmetric. The symmetric equilibrium bidding strategies are

\[ \beta(s) = \mathbb{E}[v(x,x) | x \leq s]. \] (28)

Using these bidding strategies, the expected payoff of the target from the auction is

\[ \mathbb{E}[\max(\beta(s_1), \beta(s_2))] = 2 \int_0^1 \int_0^s v(x,x) \, dx \, ds. \] (29)

Because the target knows that in the common-value case bidders will never approach it, it will initiate the auction and solicit bids if and only if (29) exceeds \( I \). The next proposition summarizes the equilibrium.

**Proposition 3.** Let

\[ \hat{I}^{cv} \equiv 2 \int_0^1 \int_0^s v(x,x) \, dx \, ds. \] (30)

If \( I < \hat{I}^{cv} \), the target initiates the auction at the initial date, and the bidders bid according to symmetric strategies (28). If \( I > \hat{I}^{cv} \), the target does not initiate the auction, no bidder approaches the target, and the takeover never happens.
4.2 The Private-Value Case

Now, consider the optimal decision by the target to initiate the contest in the private-value model. We solve the game by backward induction. First, suppose that the target chooses not to initiate the auction at the initial date. In the next period, it is discovered by one of the bidders with probability \(2p\), and conditional on discovery, it is approached by the bidder with probability \(1 - \hat{s}\), where \(\hat{s}\) is the marginal type that approaches the target, given by (23). Conditional on being approached by a bidder, the expected revenues of the target from the auction are

\[
\int_{b}^{b} bd \left( \Pr \left( b_{1} \leq b \Pr \left( b_{2} \leq b \right) \right) \right) \quad (31)
\]

\[
= \int_{b}^{b} bd \left( \Pr \left( s_{1} \leq f_{1} (b) \Pr \left( s_{2} \leq f_{2} (b) \right) \right) \right) \quad (32)
\]

\[
= \int_{b}^{b} bd \left( f_{1} (b) f_{2} (b) \right), \quad (33)
\]

where \(b\) under the integral is the winning bid. Therefore, the continuation value of the target from not initiating the auction today is

\[
2p (1 - \hat{s}) \int_{b}^{b} bd \left( f_{1} (b) f_{2} (b) \right). \quad (34)
\]

Second, suppose that the target chooses to initiate the auction at the initial date. Because the auction is target-initiated, both bidders are symmetric. Let \(\gamma (s)\) denote the bid of a bidder with signal \(s\). A standard argument implies that in equilibrium each bidder bids the expected value to his competitor conditional on winning:

**Lemma 4 (equilibrium in the target-initiated PV auction).** If the target initiates the auction, the two bidders are symmetric. The symmetric equilibrium bidding strategies are

\[
\gamma (s) = \mathbb{E} [v (x) | x \leq s]. \quad (35)
\]
Using these bidding strategies, the expected payoff of the target from the auction is

$$
\mathbb{E} [\max (\gamma (s_1), \gamma (s_2))] = 2 \int_0^1 \int_0^s v(x) \, dx \, ds. \tag{36}
$$

Combining these two cases, the target chooses to initiate the auction if and only if

$$
2 \int_0^1 \int_0^s v(x) \, dx \, ds - I \geq 2p (1 - \hat{s}) \int_0^b bd (f_1(b) f_2(b)). \tag{37}
$$

Inequality (37) illustrates the trade-off that the target faces when deciding whether to initiate the auction herself or not. By not initiating the auction herself, the target saves on cost $I$ and may obtain higher expected revenues, if it is approached by bidders. At the same time, she risks that she may not be approached by bidders either because potential bidders do not discover the target or discover it but have low enough valuations.

The next proposition summarizes the equilibrium of the whole game.

**Proposition 4.** Let

$$
\hat{I}^{pv} \equiv 2 \int_0^1 \int_0^s v(x) \, dx \, ds - 2p (1 - \hat{s}) \int_0^b bd (f_1(b) f_2(b)). \tag{38}
$$

If $I < \hat{I}^{pv}$, then the target initiates the auction at the initial date, and the bidders bid according to symmetric strategies (35). If $I > \hat{I}^{pv}$, then the target waits to be approached by bidders. In this case, a bidder discovering the target approaches her if and only if his signal exceeds $\hat{s}$, given by (23).

### 5 Model Implications

In this section, we discuss implications of the model.
5.1 Efficiency of Takeovers and the Role for Shareholder Activism

The model implies that bidders are strongly disincentivized to approach targets in common-value environments. What are the situations that represent these environments? On a very high level, motives for acquisitions can be divided into two groups. First, acquisitions can be motivated by synergies of a target with a particular bidder, such as potential gains from combining technologies. Because high synergies with one bidder do not necessarily mean high synergies with the other bidder, acquisitions driven by synergy motives are closer to the private-value setting. Second, acquisitions can be motivated by poor performance of the incumbent management of the target. Because gains from replacing bad management and/or changing policies are likely to be similar for different bidders, who, however, may have different estimates of the value created, such environments are closer to the common-value setting. Thus, the paper implies that it is precisely the second type of targets that potential bidders have little incentives to initiate.

Assuming for a moment that inefficiently managed targets are represented well by the pure common-values setting, the only way for such deals to take place is to be initiated by the target. If, however, the management is entrenched and wants to preserve independence, the target has little incentive to initiate the deal. As a consequence, inefficiently-run companies can remain without a change in ownership for a long time, even if gains from the change in ownership are substantial. Thus, the role of takeovers as a corporate governance mechanism can be limited. In contrast to the free-rider problem in tender offers that applies uniformly to both common- and private-value takeover bids in the form of tender offers, the problem that we emphasize is centered around common-value takeovers; the effect is limited in private-value settings, such as when synergies are bidder-specific.

The lack of incentive for bidders to initiate common-value auctions gives rise to alternative ways of promoting takeovers. In particular, it gives rise to shareholder activism. Consider the target with an extremely entrenched management, such that it never wants to put itself for sale no matter how high the potential economic surplus from the sale is. However, the target can be forced to put itself for sale if the board is pressured by a blockholder,
such as an activist hedge fund. In this context, one think about cost $I$ as the cost that a shareholder (or a potential shareholder) needs to incur to convince the board of the target to sell itself. A case in point is the acquisition of Blue Coat by Thoma Bravo, discussed in the introduction. In that case, an activist hedge fund Elliot Associates accumulated a 9% ownership stake in Blue Coat and forced the board of Blue Coat to auction the company off. Interestingly, toeholds can also incentivize the bidders to approach the target. An extension of the model in Section 7 studies this question in detail.

While the model focuses on a single target, it is straightforward to apply its results for the problem of selecting one target out of a multitude of potential targets by a bidder. Because of unraveling of initiation in common-value auctions, the model implies that bidders will tend to approach targets, in which they have a substantive private value component of valuation, even if other potential targets have considerably greater potential gains from the acquisition.

5.2 Empirical Predictions about Initiation

Because the degree of the common-value component in the valuation is not directly observable, it needs to be proxied in empirical tests. A potential proxy is whether the takeover battle is between strategic bidders or financial (private equity) bidders. Intuitively, because different private equity firms tend to use similar strategies after they acquire the target, their valuations should have a significant common component, even though they may have different estimates of it. Given this, the model yields the following predictions:

1. Contests among financial bidders are more likely to be target-initiated than contests among strategic bidders.

2. In bidder-initiated PE deals, the initiating bidder is likely to have a toehold.

3. A strategic bidder approaches targets in which she has a high private component of the valuation (even if synergies are lower)
The first prediction is a direct consequence of the model. It is consistent with the summary statistics on parties initiating takeovers provided by Fidrmuc et al. (2012). The second prediction follows from the result that a toehold allows the initiating bidder to preserve part of the surplus, even if his type is the lowest. Hence, a toehold can help bidders initiate common-value contests. We formally model toeholds in Section 7. The third prediction directly follows from the discussion in Section 5.1.

5.3 Empirical Predictions about Bidding

The model yields a number of implications about bidding in bidder- versus target-initiated takeover auctions. There are substantial differences which stem from endogenous initiation of takeovers by bidders and the resulting asymmetries between the initiating bidder and his competitors. These asymmetries are absent if targets initiate contests themselves.

1. All else equal, bidders in target-initiated auctions are weaker (have lower valuations) than bidders in bidder-initiated auctions.

2. Conditional on the same valuations, bidders bid less aggressively in target-initiated deals.

3. In bidder-initiated deals, the initiating bidder is stronger (in the sense of the first-order stochastic dominance of the distribution of valuations) than the other bidders.

4. In bidder-initiated deals, conditional on the same valuation, the non-initiating bidder bids more aggressively than the initiating bidder: $\gamma_2(s) > \gamma_1(s)$ for any $s < 1$.

5. In bidder-initiated deals, unconditionally on the exact valuation, the initiating bidder bid more aggressively and wins more often: $\mathbb{E}[\gamma_1(s_1)] > \mathbb{E}[\gamma_2(s_2)]$.

All predictions are novel and testing them is potentially an interesting area of future research.
6 A Dynamic Model

To capture the target’s incentives to initiate the takeover, Section 4 relied on a particular timeline in which the strategic motive of the bidders to wait was shut down. In this section, we show that our results on target versus bidder initiation are not driven by a particular choice of the timeline. We allow all agents to optimally initiate the takeover at any point of time. The complication is that now everyone updates their beliefs about the bidders’ types following not only the choice of initiation but also the choice of inaction. To make this learning tractable and intuitive, we consider the dynamic initiation problem in the model with two types of bidders.

Time is continuous. The discount rate in the economy is $r$. Over an infinitesimal time interval $[t, t + dt]$, each bidder independently discovers the target with probability $\lambda dt$. When bidder $i$ discovers the target, he observes a private signal $s_i$ about his valuation of the target. Bidders’ signals are independent and come from a binary distribution: $s_i = 1$ with probability $p_0$ and $s_i = 0$ with probability $1 - p_0$. We refer to a bidder who obtained signal $s_i = 1$ ($s_i = 0$) as the high (low) type.

After the bidder discovers the target, he can approach it thereby putting it “in play”. To approach the target, bidder $i$ needs to incur a small positive cost $c$. To simplify exposition, in this section $c$ is just above zero. This prevents bidders of low but not high type from initiating the takeover. If the target is put in play and bidder $j$ has not discovered it earlier, he immediately learns about the target’s existence and observes a private signal $s_j$. The rest of the model setup is the same as in Section 2.

Assume first that bidders only can initiate the takeover contest. Observe that a low type can never obtain a positive surplus from the auction, so he never initiates the contest. As a consequence, if the target is approached by the bidder, both the target and the rival learn that the initiating bidder’s type is high.

Consider the target approached by bidder 1 at date $t$. Let $p(t)$ denote the belief of

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8 When distribution of bidders’ types is continuous, a dynamic model has multiple equilibria that can only be studied numerically.

9 Similar to Section 2, any positive $c$ results in unraveling in common-value auctions.
the target and bidder 1 about the probability that the type of bidder 2 is high. It is straightforward to show that if \( p_0 \in (0, 1) \) then \( p(t) \in (0, 1) \). At the auction stage, bidders are thus asymmetrically informed. Bidder 1 knows \( s_1 \), but not \( s_2 \). In contrast, bidder 2 knows \( s_2 \) but also infers \( s_1 = 1 \) from observing that bidder 1 initiated the takeover contest.

Propositions 5 and 6, similarly to Propositions 1 and 2, summarize the unique equilibrium in, correspondingly, the common-value and private-value auction:

**Proposition 5.** Consider a common-value auction. In the unique Bayes-Nash equilibrium, a bidder never approaches the target after he discovers it. The off-equilibrium beliefs are such that if a bidder approaches the target, his type is high.

**Proposition 6.** Consider a private-value auction. Suppose that \( p(t) < r/\lambda \). Then, in the unique Bayes-Nash equilibrium, a bidder of high type, \( s_i = 1 \), who discovers the target at time \( t \), approaches the target immediately, while a bidder of low type, \( s_i = 0 \), never approaches the target.

Suppose that \( p(t) > r/\lambda \). Then, in the unique Bayes-Nash equilibrium, a bidder of high type, \( s_i = 1 \), who discovers the target at time \( t \), randomizes between approaching it immediately and later. The randomization is such that the implied probability that the target is approached over a small period \([t, t + dt]\) is \( r/p(t)dt \). A bidder of low type, \( s_i = 0 \), never approaches the target.

Intuitively, the delay by bidder 1 happens when the probability that the rival is of high type, \( p(t) \), is high. In this case, \( p(t) \) decreases the fastest with time. As a result, for high \( p(t) \), an increase in the expected surplus that bidder 1 achieves by waiting and confirming that his competitor is more likely to be of low type outweighs costs of delay captured by the discount rate.

Next, as in Section 4, we consider the case when both the target and the bidders can
initiate the contest at any time. As before, the target has to pay a fixed cost $I$ to initiate the contest herself. Proposition 7, similarly to Proposition 3, establishes that common-value contests can only be initiated by the target:

**Proposition 7.** Consider a common-value auction. If $I < \hat{I}^{cv}$, where $\hat{I}^{cv}$ are expected revenues of the target in a symmetric first-price auction, then in the unique Bayes-Nash equilibrium, the target initiates the auction immediately at date 0. If $I > \hat{I}^{cv}$, the target does not initiate the auction, no bidder approaches the target, and the takeover never happens. The off-equilibrium beliefs are such that if a bidder approaches the target, his type is high.

The proof is analogous to that of Proposition 3. Next, consider the optimal decision by the target to initiate the contest in the private-value model. Consider date $t$. If the target initiates the auction immediately, its expected surplus in the auction with symmetrically informed bidders is

$$v(0) + p^2 (v(1) - v(0)) - I,$$

where, for simplicity of notation, $p = p(t)$.\(^{10}\) Next, we calculate expected revenues of the target when it is approached by a bidder. The expected surplus of the initiating bidder is

$$(1 - p) (v(1) - v(0)).$$

The expected surplus of bidder 2 is zero, if he is of low type, $s_2 = 0$, and $(1 - p) (v(1) - v(0))$, if he is of high type, $s_2 = 1$. Hence, the expected surplus of both bidders is

$$
(1 - p) (v(1) - v(0)) + p (1 - p) (v(1) - v(0)) \\
= (1 + p) (1 - p) (v(1) - v(0)).
$$

\(^{10}\)The easiest way to see this is to recall the Revenue Equivalence Principle which states that, under the conditions of the model, the target’s expected surpluses in the first-price and second-price auction are equal. In the second-price auction, the only case the target receives the payoff $v(1)$ instead of $v(0)$ is when both bidders are of high type, which happens with probability $p^2$. 

29
The total surplus to both bidders and the target is \( v(1) \). Hence, the expected revenues of the target are

\[
v(1) - (1 - p^2)(v(1) - v(0)) = v(0) + p^2(v(1) - v(0)) \equiv S(p)
\]

(42)

When the target decides on the optimal initiation, the trade-off is as follows. On one hand, it takes time to wait to be approached by the bidder. On the other hand, the target need not spend cost \( I \). The expected surpluses from bidder- and target-initiated contests also dynamically change as the probability that either bidder is of high type is updated following their actions or inaction.

Let \( R(p) \) denote the expected surplus of the target under the optimal policy assuming that \( p(t) < r/\lambda \): each bidder approaches the target immediately once he discovers it and receives a high-type signal. Proposition 8 establishes the conditions under which the target will prefer to initiate the contests in the private-value case:

**Proposition 8.** Consider a private-value auction. Consider date \( t \). If

\[
rS(p) + \lambda p(1 - p)S'(p) - 2\lambda pI \geq rI,
\]

(43)

where \( p \equiv p(t) \) denotes the belief of the target that both bidders’ type is high, then it is optimal for the target to initiate the takeover contest immediately. Otherwise, initiation is delayed until either a bidder approaches the target or \( p \) becomes sufficiently low for (43) to hold.

Equation (43) is obtained in the Appendix from the dynamics of the optimal expected surplus of the target, \( R(p) \), as a function of belief \( p \). Let \( \tilde{p} \) be the indifference point at which (43) holds with equality. When \( \tilde{p} \) is close to zero, the left-hand side of (43) approaches \( rv(0) \). Assuming that \( v(0) > I \) (i.e., it is better to initiate and be acquired by a low-value...
The equilibrium initiation strategies in the private-value auction: Cases I and II. This and the next figure show the equilibrium initiation strategy for the target as a function of the belief that one of the bidders is of the high type, \( p(t) \), for four different sets of parameters.

In general, the optimal initiation policy of the target is non-monotonic in \( p(t) \). Four different cases are shown on Figures 4 and 5. In the case shown in the left panel of Figure 4, \( I \) is low so the target finds it optimal to initiate the takeover contest for any \( p(t) \). Thus, in this case, the takeover contest occurs immediately and is target-initiated. In the right panel of Figure 4, \( I \) is higher, so the optimal policy of the target is given by a threshold: it initiates the takeover contest if \( p(t) \) is above a certain threshold and does not if \( p(t) \) is below a certain threshold when takeover gains do not outweigh initiation costs. Thus, in this case, if \( p_0 \) is high enough, the takeover contest occurs immediately and is target-initiated; if \( p_0 \) is below a certain threshold, the target waits until it is approached by a bidder, so the takeover contest is either bidder-initiated or never occurs.

In the two cases shown on Figure 5, the optimal policy of the target is not monotone
Figure 5: The equilibrium initiation strategies in the private-value auction: Cases III and IV. This and the previous figure show the equilibrium initiation strategy for the target as a function of the belief that one of the bidders is of the high type, $p(t)$, for four different sets of parameters.

In $p(t)$. In the left panel, the optimal policy of the target is to initiate the contest if $p(t)$ is high or low and to wait, if $p(t)$ is intermediate. Thus, if $p_0$ is intermediate, the target initially waits until it is approached by a bidder, and if it is not approached for some time, it preempt bidders and initiates the takeover itself. This happens once $p(t)$ decreases to the lowest point of the “no target initiation” interval. Finally, the right panel of Figure 5 shows a case of high $I$ in which the target postpones initiation also for very high values of $p(t)$.$^{11}$

$^{11}$Cases III and IV to some extent resemble takeover contests in industries with high asymmetry of information, e.g., hi-tech industries. Initiation costs are relatively high there as target-initiated contests can be interpreted as a signal of overvaluation. A common pattern in such industries is as follows: if a high-tech company is not approached by potential bidders early in its life, it can try to initiate the acquisition itself.
7 Other Extensions

7.1 Costly Information Acquisition

One may doubt whether our result on unraveling of bidder-side initiation in common-value contests extends into a setting in which the second bidder can only acquire information at a cost. For example, in a different setting, Fishman (1988) shows that if a subsequent bidder can only acquire information about his valuation at a cost, the first bidder can preempt him from participating in the auction by submitting a high enough bid. One may worry that a similar preemption can arise in our model, resulting in profitable initiation strategies.

In this section, we show that this concern is unwarranted. The reason is that even if the second bidder finds it too costly to acquire information, he can never obtain a lower surplus from participating in the auction than the initiating bidder (or the initiating bidder of the lowest type in the model with continuous types), because the second bidder learns information about the common value from the decision of the first bidder to approach the target.

To establish this result, we incorporate the information acquisition setup of Fishman (1988) into our common-value setting. Suppose, as in Section 6, that bidder $i$ “discovers” the target with probability $\lambda dt$ over an infinitesimal time interval and learns his signal $s_i$. Then, bidder $i$ can make an offer to acquire the target. Having observed the offer of bidder $i$, bidder $j$ decides if he is willing to pay cost $c$ to learn his signal $s_j$. Then, bidder $j$ submits his bid, and bidder $i$ can increase his bid, if he wants, simultaneously with bidder $j$’s bid.

The above intuition of why the results of the common-value model remain unchanged in the presence of information acquisition costs is formalized in the following proposition.

**Proposition 9.** In the unique Bayes-Nash equilibrium of the model with information acquisition costs, a bidder never approaches the target after he discovers it. The off-equilibrium beliefs are such that if a bidder approaches the target, his type is high.
7.2 Toehold Acquisition

Consider a dynamic model of Section 6. Suppose that prior to approaching the target, bidder 1 can secretly acquire toehold \( \theta < 0.5 \) at price \( T(\theta, p) \) where \( p \equiv p(t) \) denotes the belief of the target or its shareholders that both bidders’ type is high. We are primarily interested in the common-value case. Consider the bidding stage at which bidder 1 has already acquired the toehold and approaches the target. Suppose that bidder 2 does not acquire the toehold. If bidder 1 acquires the remaining shares at price \( b \), the payoff to bidder 1 is \( v(1, s_2) - (1 - \theta) b \) and the payoff to bidder 2 is zero. If bidder 2 acquires the target at price \( b \), the payoff to bidder 1 is \( \theta b \) and the payoff to bidder 2 is \( v(1, s_2) - b \). One can see that a positive toehold, by lifting the payoff of bidder 1 in case of loss, can help resolve the unraveling problem in the common-value case.

Proposition 10 derives the expected surplus of the initiating bidder and formalizes his incentives to initiate the takeover contest:

**Proposition 10.** The expected surplus of bidder 1 depends on the expected bid by bidder 2: \( \theta \mathbb{E}[b_2] \). The choice of bidder 1 to acquire the toehold and initiate the contest depends on the difference \( \theta \mathbb{E}[b_2] - T(\theta, p) \). In particular, if the acquisition is not expected by the market, \( T(\theta, p) = 0 \) and bidder initiation can be optimal.

The intuition is as follows. As in Bulow, Huang, and Klemperer (1999), the toehold of bidder 1 leads to more aggressive bidding of both bidder 1 and bidder 2 of the high type. As a result, the expected payoff of bidder 2 goes down. The expected payoff of bidder 1 goes up if and only if the expected bid of bidder 2 exceeds the price at which bidder 1 acquired the toehold. This is clearly the case when the acquisition is not expected by the market, i.e., if the target is selected at random from a large pool of companies.
8 Conclusion

In this paper, we show how bidders’ valuations of target’s assets affect their and the target’s decisions to initiate takeover contests. The decision to approach the target sends a signal about the initiating bidder’s information about its valuation of the target. In private-value contests, e.g., competition between strategic bidders, the released information does not affect the rival’s valuation. As a result, bidders with good information about the target always initiate the contest unless they are pre-empted by the target who puts itself for sale earlier. In common-value contests, e.g., competition between financial bidders, the released information is useful for the rival, which erodes the initiating bidder’s revenues. As a result, bidders never initiate common-value contests but are ready to participate in target-initiated competitive processes as those do not disclose the bidders’ signals. This result is robust to whether we consider binary or continuous distribution of the bidders’ signals about the target, and to the introduction of endogenous signal acquisition costs. However, if a common-value bidder is able to obtain a toehold in the target at a low price, he may also find it optimal to initiate the takeover contest. We provide implications for the dynamics of the corporate control market, both on aggregate and individual firm level. In particular, we derive conditions under which deals can be bidder- or target-initiated, or not initiated at all. We also characterize the equilibrium bids and the division of surplus among the target, the initiating bidder, and the non-initiating bidders.

Appendix

Special cases

Example of the common-value setting: \( v(s_1, s_2) = \frac{1}{2} (s_1 + s_2) \). To find the equilibrium, let \( \phi_1(b, \hat{s}) = \alpha_i + \beta_i b \). Plugging in to the boundary conditions and differential equations yields The first boundary condition is:

\[
\hat{s} = \alpha_1 + \beta_1 \frac{1}{2} \hat{s},
\]

\[
0 = \alpha_2 + \beta_2 \frac{1}{2} \hat{s}.
\]
\[
\dot{s} = \alpha_1 + \alpha_2 + (\beta_1 + \beta_2) \frac{\dot{s}}{2}
\]

The second boundary condition:

\[
\begin{align*}
1 &= \alpha_1 + \beta_1 \bar{b} \\
1 &= \alpha_2 + \beta_2 \bar{b} \\
2 &= \alpha_1 + \alpha_2 + (\beta_1 + \beta_2) \bar{b}
\end{align*}
\]

Plugging into the differential equation yields:

- Bidder 1’s strategy:
  \[
  \phi_1(b, \dot{s}) = \frac{\dot{s}^2}{2 - \dot{s}} + \frac{4(1 - \dot{s})}{2 - \dot{s}} b;
  \]

- Bidder 2’s strategy:
  \[
  \phi_2(b, \dot{s}) = -\frac{2\dot{s}}{2 - \dot{s}} + \frac{4}{2 - \dot{s}} b.
  \]

For both strategies, \( b \in [\frac{1}{2} \dot{s}, \frac{\dot{s} + 2}{4}] \). The bidding strategies given signals are inverses of \( \phi_1(b) \) and \( \phi_2(b) \):

\[
\begin{align*}
\beta_1(s, \dot{s}) &= -\frac{\dot{s}^2}{4(1 - \dot{s})} + \frac{2 - \dot{s}}{4(1 - \dot{s})} s \\
\beta_2(s, \dot{s}) &= \frac{\dot{s}}{2} + \frac{2 - \dot{s}}{4} s.
\end{align*}
\]

**Example of the private-value setting:** \( v(s_i) = s_i \). First, let us find type \( \dot{s} \). The payoff of the indifferent type \( \dot{s} \) is

\[
P_1(\gamma_1(\dot{s}, \dot{s}), \dot{s}, \dot{s}) = \max_b b (\dot{s} - b) > 0,
\]

which implies \( \gamma_1(\dot{s}, \dot{s}) = \dot{s}/2 \) and \( P_1(\gamma_1(\dot{s}, \dot{s}), \dot{s}, \dot{s}) = \dot{s}^2/4 \). Therefore, \( \dot{s} = 2\sqrt{c} \).

Next, consider the system of differential equations determining optimal inverse bidding functions:

\[
\begin{align*}
 f'_2(b) &= \frac{f_2(b)}{f_1(b) - \dot{s}} \\
 f'_1(b) &= \frac{f_1(b) - \dot{s}}{f_2(b) - \dot{s}}.
\end{align*}
\]

They are equivalent to

\[
\begin{align*}
(f'_2(b) - 1) (f_1(b) - b) &= f_2(b) - f_1(b) + b, \\
(f'_1(b) - 1) (f_2(b) - b) &= f_1(b) - \dot{s} - f_2(b) + b.
\end{align*}
\]
Adding the two equations yields
\[
\frac{d}{db} [(f_1(b) - b) (f_2(b) - b)] = 2b - \hat{s}.
\] (47)

Integrating, we obtain
\[
(f_1(b) - b) (f_2(b) - b) = C + b^2 - \hat{s}b.
\]
This equation holds for any bid \( b \in [g, \hat{g}] \). In particular, it holds for \( b = g = \hat{s}/2 \). Substituting and using the boundary condition \( f_2(\hat{g}) = g \), we obtain
\[
C = \frac{1}{4} \hat{s}^2.
\]

Plugging back into differential equations yields:
\[
f'_1(b) = \frac{(f_1(b) - \hat{s}) (f_1(b) - b)}{(b - \frac{\hat{s}}{2})^2},
\]
\[
f'_2(b) = \frac{f_2(b) (f_2(b) - b)}{(b - \frac{\hat{s}}{2})^2}.
\]
Finally, since \( f_1(\bar{b}) = f_2(\bar{b}) = 1 \),
\[
\bar{b} = \frac{1 - \frac{1}{4} \hat{s}^2}{2 - \hat{s}}.
\]

**Additional Proofs**

**Proof of Lemma 4.** The expected payoff of a bidder with signal \( s \) who bids \( b \) is
\[
P(b, s) = \gamma^{-1}(b) (v(s) - b).
\] (48)

The first-order condition is
\[
\frac{\partial \gamma^{-1}(b)}{\partial b} (v(s) - b) - \gamma^{-1}(b) = 0.
\] (49)

In equilibrium, the maximum must be reached at \( b = \gamma(s) \). Therefore,
\[
s \gamma'(s) = v(s) - \gamma(s).
\] (50)

Equivalently,
\[
\frac{d(s \gamma(s))}{ds} = v(s)
\] (51)
This differential equation is solved subject to the initial value condition that the lowest type bids his value, $\gamma(0) = v(0)$. Therefore,

$$
\gamma(s) = v(0) + \frac{1}{s} \int_0^s v(x) \, dx \\
= \mathbb{E}[v(x) \mid x \leq s]. 
$$

(52)

In other words, the bidder bids the expected value of his competitor, conditional on winning.

**Lemma 5 (equilibrium in the bidder-initiated CV auction of a dynamic model).** Suppose bidder 1 of high type, $s_1 = 1$, approaches the target at date $t$, initiating the auction. The bidders’ strategy profiles in the unique equilibrium are as follows:

- If bidder 2’s type is low, $s_2 = 0$, she bids $v(1, 0)$. If bidder 2’s type is high, $s_2 = 1$, she employs a mixed strategy bidding over the support $[v(1, 0), v(1, 0) + p(t)(v(1, 1) - v(1, 0))]$ (53) according to distribution

$$
F_{2,1}(b) = \frac{1 - p(t)b - v(1, 0)}{p(t)v(1, 1) - b}; 
$$

(54)

- Bidder 1 employs a mixed strategy bidding over the support $[v(1, 0), v(1, 0) + p(t)(v(1, 1) - v(1, 0))]$ (55) according to distribution

$$
F_1(b) = (1 - p(t)) \frac{v(1, 1) - v(1, 0)}{v(1, 1) - b}. 
$$

(56)

The expected surplus of bidder 2 is zero, if $s_2 = 0$, and $(1 - p(t))(v(1, 1) - v(1, 0))$, if $s_1 = 1$. The expected surplus of bidder 1 is zero.

**Proof of Lemma 5.** First, note that an equilibrium in pure strategies does not exist. Intuitively, if all bidders submitted pure-strategy bids, a high-type bidder could increase his surplus by outbidding other bidders by an infinitesimal amount. Suppose bidder 1 bids distribution $F_1(b)$ and bidder 2 of high type bids distribution $F_{2,1}(b)$. If bidder 2 of high type bids $b$, she wins with probability $F_1(b)$, and in case she wins, she obtains valuation $v(1, 1)$ and pays her bid $b$. Therefore, her expected surplus from bidding $b$ is

$$
F_1(b)(v(1, 1) - b). 
$$

(57)
Taking the first-order condition:

\[ b = v(1, 1) - \frac{F_1(b)}{f_1(b)}, \]  

(58)

where \( f_1(b) = F_1'(b) \). To play mixed strategies, this equation must hold for different \( b \).

Solving this differential equation:

\[ F_1(b) = \frac{C_1}{v(1, 1) - b}, \]  

(59)

where \( C_1 \) is some constant.

Next, note that bidder 2 of low type in equilibrium must bid her expected valuation \( v(1, 0) \). Consider now bidder 1 submitting a bid \( b > v(1, 0) \). If she competes against the low type, she wins with probability one, obtains valuation \( v(1, 0) \), and pays \( b \). If she competes against the high type, she wins with probability \( F_{2,1}(b) \), and in case she wins, she obtains valuation \( v(1, 1) \) and pays her bid \( b \). Thus, the expected surplus of bidder 1 is

\[ pF_{2,1}(b) (v(1, 1) - b) + (1 - p) (v(1, 0) - b). \]  

(60)

Taking the first-order condition:

\[ b = v(1, 1) - \frac{pF_{2,1}(b) + 1 - p}{pf_{2,1}(b)}. \]  

(61)

Solving this differential equation:

\[ F_{2,1}(b) = \frac{C_2}{v(1, 1) - b} - \frac{1 - p}{p}. \]  

(62)

To pin down constants \( C_1 \) and \( C_2 \) note that bidder 1 would never bid below \( v(1, 0) \), which is the lowest value she can possibly obtain. Hence, bidder 2 of high type would also never bid below \( v(1, 0) \). Thus, \( C_2 \) is pinned down by condition \( F_{2,1}(v(1, 0)) = 0 \):

\[ C_2 = \frac{1 - p}{p} (v(1, 1) - v(1, 0)). \]  

(63)

Thus, bidder 2 of high type mixes \( b \in [v(1, 0), v(1, 0) + p(v(1, 1) - v(1, 0))] \) according to distribution

\[ F_{2,1}(b) = \frac{1 - p}{p} \frac{b - v(1, 0)}{v(1, 1) - b}. \]  

(64)

Constant \( C_1 \) can be found from the condition that bidder 1 can never find it optimal to bid above \( v(1, 0) + p(v(1, 1) - v(1, 0)) \). Intuitively, under no circumstances, bidder 2 bids above this amount, so if bidder 1 bid above this amount, she could increase her surplus by deviating and bidding marginally less. Thus, \( C_1 \) is determined by condition
\[ F_1(v(1,0) + p(v(1,1) - v(1,0))) = 1: \]

\[ C_1 = (1 - p)(v(1,1) - v(1,0)) . \]  \hfill (65)

Thus, bidder 1 bids \( v(0) \) with probability \( 1 - p \) and bids \( b \in (v(0), v(1,0) + p(v(1,1) - v(1,0))) \) according to distribution

\[ F_1(b) = (1 - p) \frac{v(1,1) - v(1,0)}{v(1,1) - b} . \]  \hfill (66)

Clearly, the expected surplus of the low-type is zero. Because of mixed strategies, the expected surplus of high types can be calculated by plugging in \( b = v(1,0) \) in their payoff functions. The expected surplus of bidder 1 is

\[ pF_{2,1}(v(1,0))(v(1,1) - v(1,0)) + (1 - p)(v(1,0) - v(1,0)) = 0. \]  \hfill (67)

Thus, bidder 1 obtains zero surplus in expectation. The expected surplus of bidder 2 of high type is

\[ F_1(v(1,0)) (v(1,1) - v(1,0)) = (1 - p)(v(1,1) - v(1,0)) > 0. \]  \hfill (68)

**Proof of Proposition 5.** Clearly, no bidder approaching the target and the off-equilibrium belief that a bidder approaching the target has a high type is an equilibrium, as approaching the target results in the same zero surplus that a bidder obtains in equilibrium. Let us show that this equilibrium is unique. Suppose that the off-equilibrium belief that the approaching bidder’s type is high is less than one, \( \Pr[s_1 = 1 | \text{approach}] < 1 \). Then, the high type obtains a positive surplus in expectation from approaching the target. Because the costs of approaching the target are infinitesimal, the high-type would deviate from the strategy of never approaching the target: The strategy of never approaching the target yields zero in equilibrium under the belief that no other bidder approaches the target, while a deviation results in a positive expected payoff.

**Lemma 6 (equilibrium in the bidder-initiated PV auction of a dynamic model).** Suppose bidder 1 of high type, \( s_1 = 1 \), approaches the target at date \( t \), initiating the auction. The bidders’ strategy profiles in the unique equilibrium are as follows:

- If bidder 2’s type is low, \( s_2 = 0 \), she bids \( v(0) \). If bidder 2’s type is high, \( s_2 = 1 \), she employs a mixed strategy bidding over the support

\[ [v(0), v(0) + p(t)(v(1) - v(0))] \]  \hfill (69)
according to distribution

\[ G_{2,1}(b) = \frac{1 - p(t) b - v(0)}{p(t) - v(1) - b}; \quad (70) \]

- Bidder 1 employs a mixed strategy bidding over the support

\[ [v(0), v(0) + p(t)(v(1) - v(0))] \quad (71) \]

according to distribution

\[ G_1(b) = (1 - p(t)) \frac{v(1) - v(0)}{v(1) - b}. \quad (72) \]

The expected surplus of bidder 2 is zero, if \( s_2 = 0 \), and \((1 - p(t))(v(1) - v(0))\), if \( s_1 = 1 \). The expected surplus of bidder 1 is \((1 - p(t))(v(1) - v(0))\).

**Proof of Lemma 6.** Our first observation is that an equilibrium in pure strategies does not exist. Intuitively, a rival would want to outbid its competitor by an infinitesimal amount. Suppose bidder 1 bids distribution \( G_1(b) \). Bidder 2 of type \( s_2 = 0 \) can never win in equilibrium. Any bid \( b_{2,0} < v(0) \) will be an equilibrium bid. Suppose bidder 2 bids distribution \( G_{2,1}(b) \). His expected surplus from bidding \( b \)

\[ G_1(b)(v(1) - b). \quad (73) \]

Taking the first-order condition with respect to \( b \):

\[ b = v(1) - \frac{G_1(b)}{g_1(b)}, \quad (74) \]

where \( g_1(b) = G'_1(b) \). To play mixed strategies, this equation must hold for different \( b \). Solving this differential equation:

\[ G_1(b) = \frac{D_1}{v(1) - b}, \quad (75) \]

where \( D_1 \) is some constant.

Consider now bidder 1. His expected surplus from bidding \( b \geq v(0) \) is

\[ (pG_{2,1}(b) + 1 - p)(v(1) - b). \quad (76) \]

Taking the first-order condition with respect to \( b \):

\[ b = v(1) - \frac{pG_{2,1}(b) + 1 - p}{pg_{2,1}(b)}, \quad (77) \]
where $g_{2,1} (b) = G'_{2,1} (b)$. To play mixed strategies, this equation must hold for different $b$. Solving this differential equation:

$$G_{2,1} (b) = \frac{D_2}{v(1) - b} - \frac{1 - p}{p},$$

(78)

where $D_2$ is some constant.

The next step is to find constants $D_1$ and $D_2$. Equation (101) implies that the distribution function of bidder 1 has an atom from below. It must be the point right above the maximum possible bid of bidder 2 of type $s_2 = 0$. It is natural to select $v(0)$ as such a point. Intuitively, bidder 2 of type $s_2 = 0$ never wants to bid $v(0)$, because even if he wins, he obtains a negative surplus. At the same time, it cannot be below $v(0)$, because if it is, bidder 2 of type $s_2 = 0$ would bid above this point and would obtain a positive surplus with positive probability. But this will create incentives for bidder 1 to move his atom point further. Because the bidding schedule of bidder 1 has an atom at $v(0)$, bidder 2 of type $s_2 = 1$ would never want to bid below $v(0)$. This implies boundary condition

$$G_{2,1} (0) = 0,$$

(79)

which pins down constant $D_2$:

$$D_2 = \frac{1 - p}{p} (v(1) - v(0)).$$

(80)

Thus, the bidding schedule of bidder 2 of type $s_2 = 1$ is

$$G_{2,1} (b) = \frac{1 - p b - v(0)}{p} \frac{v(1) - v(0)}{b}.$$

(81)

The right point of the interval is given by $G_{2,1} (b) = 1$, which yields $\bar{v} = pv_1 + (1 - p) v_0$. Thus, bidder 2 never bids above $\bar{v}$. But this implies that bidder 1 never bids above $\bar{v}$ too, as otherwise, bidder 1 would reduce his bid while still winning with certainty. Hence, we obtain the boundary condition $G_1 (\bar{v}) = 1$, which pins down constant $D_1$:

$$D_1 = v(1) - \bar{v} = (1 - p) (v(1) - v(0)).$$

(82)

Thus, the size of the atom is $G_1 (v(0)) = 1 - p$.

**Proof of Proposition 6.** For simplicity of notation, let the belief of a bidder that the other bidder’s type is high, $p \equiv p(t)$. We distinguish between the two cases:
(1) Case $p \leq r/\lambda$

Suppose that at date $t$ bidder 1 discovers a target and receives signal $s_1 = 1$. If he approaches the target immediately, his expected surplus will be

$$\left(1 - p(t)\right) (v(1) - v(0)).$$

This is his expected surplus when he submits a bid $v(0)$, but it also equals to his expected surplus from any other bid that he uses in his mixed strategy.

If he delays his approaching the target for an infinitesimal time interval $dt$, his expected surplus is as follows. With probability $q(t) dt$, the target is approached by bidder 2, where $q(t)$ is to be determined. The type of bidder 2 will be high, as well as the type of bidder 1, but the type of bidder 1 will not be observed by bidder 2 when submitting bids. The expected surplus of bidder 1 will be:

$$\left(1 - p(t)\right) (v(1) - v(0)).$$

This is his expected surplus when he submits a bid $v(0)$, but it also equals to his expected surplus from any other bid that he uses in his mixed strategy. Note that this is the same surplus that bidder 1 obtains if he approaches the target today. With probability $1 - q(t) dt$, bidder 2 does not approach the target over the next instant. Let us find the conditions under which a bidder approaches the target immediately. Let $\bar{p}$ denote individually rational approaching threshold subject to the constraint that the other bidder approaches the target immediately when he discovers it. Then, the equilibrium condition is:

$$r \left(1 - \bar{p}\right) (v(1) - v(0)) = \lambda \bar{p} \left(1 - \bar{p}\right) (v(1) - v(0)).$$

Hence,

$$\bar{p} = \frac{r}{\lambda}.$$ 

Thus, for any $p_0 \leq r/\lambda$, bidders will approach the target immediately when they discover it.

(2) Case $p > r/\lambda$

There are no equilibria in pure strategies. Suppose that bidder 1 immediately approaches the target if he receives signal $s_1 = 1$. By Bayes’s rule, the evolution of $p$ is as follows:

$$dp(t) = -\lambda p(t) (1 - p(t)) dt.$$ 

Given this, approaching is individually rational if and only if $r < \lambda p$, which is inconsistent with the initial conditions on parameters $p > r/\lambda$. Suppose that under these conditions, bidder 1 never approaches the target if it is of high type. In this case, $dp = 0$, which would imply that approaching immediately is individually rational. Hence, the only possible
equilibria are in mixed strategies.

To play a mixed strategy, a bidder must be indifferent between approaching the target and postponing the decision. Let \( \lambda (p) \) denote the intensity with which a high-type bidder approaches the target at time \( t \). Bayes’ rule implies that

\[
dp(t) = -\lambda (p(t)) p(t) (1 - p(t)) \, dt. \tag{85}
\]

Let \( V (p) \) denote the expected value to the high type from the auction. In the time interval after learning the type but prior to approaching the target, \( V (p) \) satisfies

\[
(r + p\lambda (p)) V (p) = -\lambda (p) p (1 - p) V' (p) + p\lambda (p) (1 - p) (v (1) - v (0)). \tag{86}
\]

The indifference condition implies that

\[
V (p) = (1 - p) (v (1) - v (0))
\]

in the relevant time interval. As a consequence, \( V' (p) = v (0) - v (1) \). Thus, in the relevant time interval \( \lambda (p) \) satisfies

\[
r (1 - p) (v (1) - v (0)) = \lambda (p) p (1 - p) (v (1) - v (0)) \quad \Rightarrow
\]

\[
\lambda (p) = \frac{r}{p}. \tag{87}
\]

This condition must hold for all \( p(t) \) up to a certain lower threshold \( p^* \), after which all high-type bidders who discover the target approach it immediately. Threshold \( p^* \) can be determined from the clearing condition. The probability that a bidder discovers the target between time 0 and time \( \tau \) is \( 1 - e^{-\lambda \tau} \). The probability that a bidder discovers the target and is a high type is \( p_0 (1 - e^{-\lambda \tau}) \). Once \( p \) reaches \( p^* \), all these bidders need to approach the target. We split the derivation of the clearing condition into three steps.

**Step 1: Discovery of the target by high-type bidders.** In the range \([0, \tau]\), assuming \( p > p^* \) and utilizing (87), \( p \) evolves as

\[
dp(t) = -r (1 - p(t)) \, dt. \tag{88}
\]

The general solution of this differential equation is

\[
\log (1 - p) = rt + C.
\]

The boundary condition at date 0 is \( p(0) = p_0 \). Hence, \( C = \log (1 - p_0) \). As a result, we obtain \( p(t) \):

\[
\log (1 - p) = rt + \log (1 - p_0) \quad \Rightarrow
\]

\[
1 - p(t) = (1 - p_0) e^{rt} \quad \Rightarrow
\]
\[ p(t) = 1 - (1 - p_0) e^{rt}. \]

The above expression can be inverted to obtain \( e^{\alpha t} \) for any \( \alpha \) as a function of \( p \). Hence, the probability that a bidder discovers the target and is of high type by time \( t \) is

\[ p_0 (1 - e^{-\lambda t}) = p_0 \left( 1 - \left( \frac{1 - p_0}{1 - p} \right)^{\frac{1}{r}} \right). \]

**Step 2: Approaching the target.** Next, we need to derive the probability that a bidder approaches the target between 0 and \( t \), or, equivalently, between \( p_0 \) and \( p \):

\[
p_0 \int_0^t \lambda(p(s)) e^{-f(p(s))du} ds = p_0 \int_0^t \frac{r}{p(s)} e^{-f(p(s))du} ds
= p_0 \int_0^t \frac{r}{1 - (1 - p_0) e^{rs}} e^{-f(p(s))du} ds.
\]

The power of the exponential term in the integral is

\[
\int_0^s \frac{r}{1 - (1 - p_0) e^{ru}} du = \left[ ru - \log (p_0 e^{ru} - e^{ru} + 1) \right]_0^s
= rs - \log (1 - (1 - p_0) e^{rs}) + \log (p_0)
= \log \left( \frac{p_0 e^{rs}}{1 - (1 - p_0) e^{rs}} \right).
\]

Hence, the required probability is:

\[
p_0 \int_0^t \frac{r}{1 - (1 - p_0) e^{rs}} e^{-\log \left( \frac{p_0 e^{rs}}{1 - (1 - p_0) e^{rs}} \right)} ds
= p_0 \int_0^t \frac{r}{1 - (1 - p_0) e^{rs}} \frac{1 - (1 - p_0) e^{rs}}{p_0 e^{rs}} ds
= \int_0^t re^{-rs} ds = [-e^{-rs}]_0^t = 1 - e^{-rt}.
\]

**Step 3: The clearing condition.** Note that

\[ e^{-rt} = \frac{1 - p_0}{1 - p(t)}. \]

The clearing equation implies that at some \( p \), every bidder who has discovered the target and is high-type should approach it:

\[ p_0 (1 - e^{-\lambda t}) = 1 - e^{-rt}. \]
Let
\[ f(t) = p_0 \left(1 - e^{-\lambda t}\right) + e^{-rt} - 1. \]

Note that
\[
\begin{align*}
  f(0) &= 0, \\
  f'(t) &= p_0 \lambda e^{-\lambda t} - re^{-rt} \\
  &= e^{-\lambda t} \left(p_0 \lambda - r e^{(\lambda - r)t}\right), \\
  \lim_{t \to \infty} f(t) &= p_0 - 1 < 0.
\end{align*}
\]

Note that \( f'(t) > 0 \) for \( t < \tilde{t} \) and \( f'(t) < 0 \) for \( t > \tilde{t} \). By continuity, there is a unique solution \( t^* \) to \( f(t) = 0 \). Alternatively, there is a unique solution in terms of \( p(t) \) because \( p(t) \) is a monotone function of \( t \). As a function of \( p \), the market-clearing condition is
\[
p_0 \left(1 - \left(\frac{1 - p_0}{1 - p}\right)^\frac{1}{\lambda r}\right) = 1 - \frac{1 - p_0}{1 - p}.
\]

Let \( z = (1 - p_0) / (1 - p) \). We obtain:
\[
p_0 \left(1 - z^\frac{1}{\lambda r}\right) = 1 - z \quad \Rightarrow \\
p_0 z^\frac{1}{\lambda r} - z + 1 - p_0 = 0.
\]

We are interested in this equation in the range \( z \in [1 - p_0, 1] \), i.e., when \( p \in [0, p_0] \). One root of this equation is \( z = 1 \). At \( z = 1 - p_0 \), the left-hand side is
\[
p_0 \left(1 - p_0\right)^\frac{1}{\lambda r} - 1 + p_0 + 1 - p_0 = p_0 \left(1 - p_0\right)^\frac{1}{\lambda r} > 0.
\]

Consider the derivative of the left-hand side with respect to \( z \):
\[
p_0 \frac{\lambda}{r} z^\frac{1}{\lambda r} - 1.
\]

Note that \( p_0 \lambda / r > 1 \), which is our initial condition. Hence the derivative is positive at \( z = 1 \). Coupled with the inequality above and continuity of the left-hand side of the equation, this implies that the equation has at least one more root \( z \in (1 - p_0, 1) \). The second derivative is
\[
p_0 \frac{\lambda}{r} \left(\frac{\lambda}{r} - 1\right) z^\frac{1}{\lambda r} - 2 > 0,
\]

because \( \lambda / r > 1 / p_0 > 1 \). Therefore, the left-hand side is a convex function of \( p \). Hence, the
root in \( z \in (1 - p_0, 1) \) is unique. We denote it by \( z^* \), or equivalently, \( p^* \):

\[
p_0 \left( \frac{1 - p_0}{1 - p^*} \right)^2 - \frac{1 - p_0}{1 - p^*} + 1 - p_0 = 0
\]
as the unique solution in \((0, p_0)\). It is also easy to show that \( p^* < r/\lambda \).

Then, the equilibrium is as follows:

- For any \( t \) such that \( p(t) > p^* \), if a high type discovers the target, she randomizes between approaching it immediately and approaching it later. The randomization is such that the implied probability that the target is approached over a small period \([t, t + dt]\) is \( r/p(t) \ dt \);
- For any \( t \) such that \( p(t) < p^* \), if a high type discovers the target, she approaches it immediately.

The implied evolution of \( p(t) \) is

\[
dp(t) = \begin{cases} 
-r (1 - p(t)) dt, & \text{if } p(t) \geq p^*, \\
-\lambda p(t) (1 - p(t)) dt, & \text{if } p(t) < p^*.
\end{cases}
\]

**Proof of Proposition 8.** In the proof below, \( p \) denotes the belief of the target in the probability that both bidder’s type is high. Using the Bayes’ rule, the posterior probability \( p(t + dt) \) satisfies

\[
p(t + dt) = \frac{(1 - \lambda dt - p(t) \lambda dt) p(t)}{1 - 2\lambda p(t) \ dt}.
\]

We can rewrite this as

\[
\frac{p(t + dt) - p(t)}{dt} = -\lambda p(t) \frac{1 - p(t)}{1 - 2\lambda p(t) \ dt}.
\]

Taking the limit as \( dt \to 0 \),

\[
\frac{dp(t)}{dt} = -\lambda p(t) (1 - p(t)).
\]

Then, \( R(p) \) satisfies the following equation:

\[
rR(p) = -\lambda p(1 - p) R'(p) + 2\lambda p(S(p) - R(p)).
\]

Let \( \bar{p} \) denote the indifference point for the target’s decision to initiate the contest, provided that it exists. \( \bar{p} \) must satisfy the standard value-matching and smooth-pasting conditions:

\[
R(\bar{p}) = S(\bar{p}) - I,
\]

\[
R'(\bar{p}) = S'(\bar{p}).
\]
Plugging in the above two expressions into the expression for $R(p)$, we obtain:

$$r (S(\bar{p}) - I) = -\lambda \bar{p} (1 - \bar{p}) S'(\bar{p}) + 2 \lambda \bar{p} I \quad \Rightarrow \quad (95)$$

$$rS(\bar{p}) + \lambda \bar{p} (1 - \bar{p}) S'(\bar{p}) - 2 \lambda \bar{p} I = rI. \quad (96)$$

**Proof of Proposition 9.** Suppose bidder 1 approaches the target. The argument of the base model applies to show that bidder 1 with the low signal, $s_1 = 0$, can never obtain positive expected payoff from the auction, so if initiation by a bidder occurs in equilibrium, it must be a decision by the bidder with the high signal, $s_1 = 1$. Thus, if bidder 2 sees that bidder 1 approaches the target, she infers that $s_1 = 1$.

Suppose that bidder 1 makes an initial bid of $\bar{b}$. If in equilibrium bidder 2 chooses to acquire signal $s_2$, the model becomes equivalent to the base model. As shown above, the initiating bidder obtains zero surplus, which is inconsistent with equilibrium.

If in equilibrium bidder 2 chooses not to acquire signal $s_2$, her and bidder 1’s estimates of the value of the target coincide and are equal to $pv(1,1) + (1 - p) v(1,0)$. If the initial bidder 1’s bid is $\bar{b} < pv(1,1) + (1 - p) v(1,0)$, then both bidders will follow with bids $pv(1,1) + (1 - p) v(1,0)$, the asset will be allocated at random, and both bidders will obtain zero surplus. If $\bar{b} > pv(1,1) + (1 - p) v(1,0)$, then bidder 1 will acquire the target but will obtain a negative surplus, as his bid is above the value of the target. Thus, a situation in which bidder 2 never acquires a signal is also inconsistent with equilibrium.

The only remaining possibility is that bidder 2 mixes between acquiring signal $s_2$ and not acquiring it. In this case, if bidder 1 obtains a positive surplus in expectation, he must obtain it in at least one of the three possible states: when bidder 2 does not acquire the signal, when bidder 2 acquires the signal and obtains $s_2 = 0$, or when bidder 2 acquires the signal and obtains $s_2 = 1$. The latter case is clearly inconsistent with equilibrium, as it implies that bidder 2 never obtains positive surplus from the auction, which is inconsistent with her acquiring a signal at a cost. The first and second case require bidder 1 to bid below $pv(1,1) + (1 - p) v(1,0)$ and $v(1,0)$, respectively. However, if he bids in such a way in equilibrium, the optimal best response of bidder 2 is to bid above bidder 1 by a small margin. Thus, these cases are also inconsistent with equilibrium. We conclude that in common-value contests, bidder 1 can never approach the target and obtain a positive surplus even if bidder 2 must pay a cost to acquire her signal.

**Lemma 7 (equilibrium in the bidder-initiated CV auction in a dynamic model with toeholds).** Suppose bidder 1 of high type, $s_1 = 1$, who has a toehold $\theta$, approaches the target at date $t$, initiating the auction. The bidders’ strategy profiles in the unique equilibrium are as follows:

- If bidder 2’s type is low, $s_2 = 0$, she bids $v(1,0)$. If bidder 2’s type is high, $s_2 = 1$, she employs a mixed strategy bidding over the support

$$\left[ v(1,0), v(1,1) - (1 - p(t))^{1-\theta} (v(1,1) - v(1,0)) \right] \quad (97)$$
according to distribution

\[ F_{2,1}(b) = \frac{1 - p(t)}{p(t)} \left[ \frac{(v(1, 1) - v(1, 0))^{1-\theta}}{v(1, 1) - b} \right] - 1 ; \]  

(98)

- Bidder 1 employs a mixed strategy bidding over the support

\[ \left[ v(1, 0), v(1, 1) - (1 - p(t))^{1-\theta} (v(1, 1) - v(1, 0)) \right] \]  

(99)

according to distribution

\[ F_1(b) = (1 - p(t))^{1-\theta} \frac{v(1, 1) - v(1, 0)}{v(1, 1) - b}. \]  

(100)

The expected surplus of bidder 2 is zero, if \( s_2 = 0 \), and \( (1 - p(t))^{1-\theta} (v(1, 1) - v(1, 0)) \), if \( s_1 = 1 \).

**Proof of Lemma 7 and Proposition 10.** Suppose that bidder 1 bids distribution \( F_1^\theta (b) \) and bidder 2 of high type bids distribution \( F_{2,1}^\theta (b) \). It can be shown that bidder 2 of low type bids \( v(1, 0) \). If bidder 2 of high type bids \( b \), she wins with probability \( F_1^\theta (b) \), and in case she wins, her payo is \( v(1, 1) - b \). Therefore, her expected payo from bidding \( b \) is

\[ F_1^\theta (b) (v(1, 1) - b). \]

Taking the first-order condition:

\[ b = v(1, 1) - \frac{F_1^\theta (b)}{F_1^\theta (b)}, \]

where \( f_1^\theta (b) \equiv F_1^\theta (b) \). To play mixed strategies, this equation must hold for different \( b \). Solving this differential equation:

\[ F_1^\theta (b) = \frac{C_1^\theta}{v(1, 1) - b}, \]  

(101)

where \( C_1^\theta \) is some constant to be determined later.

Consider now bidder 1 submitting a bid \( b > v(1, 0) \). If he competes against the low type, he wins with probability one, and obtains the surplus of \( v(1, 0) - (1 - \theta) b \). If he competes against the high type, he wins with probability \( F_{2,1}^\theta (b) \). If he wins, he obtains the surplus of \( v(1, 1) - (1 - \theta) b \). If he loses, he obtains the surplus of \( \theta b_2 \), where \( b_2 \) is the
bid of bidder 2. Thus, the expected surplus of bidder 1 is

\[(1 - p) (v(1, 0) - (1 - \theta) b) + p F_{2,1}^\theta (b) (v(1, 1) - (1 - \theta) b) + p \left( 1 - F_{2,1}^\theta (b) \right) \theta \int_b^\infty b_2 \frac{f_{2,1}^\theta (b_2)}{1 - F_{2,1}^\theta (b_2)} db_2 \]

\[= (1 - p) (v(1, 0) - (1 - \theta) b) + p F_{2,1}^\theta (b) (v(1, 1) - (1 - \theta) b) + p \theta \int_b^\infty b_2 dF_{2,1}^\theta (b_2) . \] (102)

Taking the first-order condition:

\[b = v(1, 1) - (1 - \theta) \frac{p F_{2,1}^\theta (b) + 1 - p}{p f_{2,1}^\theta (b)} \] (103)

This differential equation is solved by

\[F_{2,1}^\theta (b) = \frac{C_{1}^\theta (v(1, 1) - b) ^{1 - \theta} - 1}{p} . \] (104)

where \(C_{1}^\theta\) is some constant. Next, we need to pin down constants \(C_{1}^\theta\) and \(C_{2}^\theta\). Bidder 1 would never bid below \(v(1, 0)\), which is the lowest valuation of the target he could possibly have. Hence, bidder 2 of high type would also never bid below \(v(1, 0)\). Thus, \(C_{2}^\theta\) is pinned down by condition \(F_{2,1}^\theta (v(1, 0)) = 0:\)

\[\frac{C_{2}^\theta (v(1, 1) - v(1, 0)) ^{1 - \theta} - 1}{p} = 0 \Rightarrow C_{2}^\theta = \frac{1 - p}{p} (v(1, 1) - v(1, 0)) ^{1 - \theta} . \]

Thus bidder 2 bids according to distribution

\[F_{2,1}^\theta (b) = \frac{1 - p}{p} \left[ \left( \frac{v(1, 1) - v(1, 0)}{v(1, 1) - b} \right) ^{1 - \theta} - 1 \right] . \] (105)

The upper boundary on the bid is given by \(F_{2,1}^\theta (\bar{b}^\theta) = 1:\)

\[1 = \frac{1 - p}{p} \left[ \left( \frac{v(1, 1) - v(1, 0)}{v(1, 1) - \bar{b}^\theta} \right) ^{1 - \theta} - 1 \right] \Rightarrow \bar{b}^\theta = v(1, 1) - (1 - p)^{\frac{1}{1 - \theta}} (v(1, 1) - v(1, 0)) \]

Without the toehold, this upper boundary is equal to

\[\bar{b} = v(1, 1) - (1 - p) (v(1, 1) - v(1, 0)) = v(1, 0) + p (v(1, 1) - v(1, 0)) . \]

Because \(\theta > 0\), \((1 - p)^{\frac{1}{1 - \theta}} < 1 - p\). Therefore, the upper boundary on the distribution
of bids of bidder 2 is higher when bidder 1 has a toehold than when he does not have a
toehold. We can extend this to the general distribution of bids in the following way:

**Lemma 8.** For any \( b > v(1,0) \), \( F_{2,1}^\theta(b) < F_{2,1}(b) \).

**Proof.** Because \( b \geq v(1,0) \),

\[
\frac{v(1,1) - v(1,0)}{v(1,1) - b} \geq 1,
\]

with the inequality being strict if \( b > v(1,0) \). Therefore, for any \( b \geq v(1,0) \),

\[
\left( \frac{v(1,1) - v(1,0)}{v(1,1) - b} \right)^{1-\theta} \leq \frac{v(1,1) - v(1,0)}{v(1,1) - b},
\]

with the inequality being strict if \( b > v(1,0) \). Hence, for any \( b > v(1,0) \),

\[
F_{2,1}^\theta(b) < F_{2,1}(b).
\]

Therefore, the toehold of bidder 1 leads to more aggressive bidding of bidder 2 of the high
type. \( Q.E.D. \)

Constant \( C_1 \) can be found from the condition that bidder 1 never finds it optimal to bid
above \( v(1,1) - (1 - p) \frac{1}{1-\theta} (v(1,1) - v(1,0)) \). Plugging into (101):

\[
\frac{C_1^\theta}{v(1,1) - v(1,1) + (1 - p) \frac{1}{1-\theta} (v(1,1) - v(1,0))} = 1 \quad \Rightarrow \quad C_1^\theta = (1 - p) \frac{1}{1-\theta} (v(1,1) - v(1,0)).
\]

Note that \( C_1^\theta < C_1 \). Thus, bidder 1 also bids more aggressively:

\[
F_1^\theta(b) = (1 - p) \frac{1}{1-\theta} \frac{v(1,1) - v(1,0)}{v(1,1) - b}. \tag{106}
\]

Bidder 1 bids \( v(1,0) \) with probability \( (1 - p) \frac{1}{1-\theta} \) and bids above this according to distribution \( F_1^\theta(b) \).

Finally, we are interested in calculating expected payoffs. The expected surplus of bidder
2 of the low type is clearly zero. Because of mixed strategies, the expected surplus of high
types of bidders 1 and 2 can be calculated by plugging in \( b = (1,0) \) in their payoff functions.
The expected surplus of bidder 1 is

\[
(1 - p) (v(1,0) - (1 - \theta) v(1,0)) + p \theta \mathbb{E}[b_2 | s_2 = 1]
\]

\[
= (1 - p) \theta v(1,0) + p \theta \mathbb{E}[b_2 | s_2 = 1]
\]

\[
= \theta \mathbb{E}[b_2].
\]
The expected surplus of bidder 2 of high type is
\[
F_1^\theta (v (1, 0)) = (1 - p)^{\frac{1}{1-\theta}} (v (1, 1) - v (1, 0)).
\]

We can see that the expected payoff of bidder 2 goes down compared to the no-toehold case because she bids more aggressively. The expected payoff of bidder 1 goes up if and only if the expected bid of bidder 2 exceeds the price at which bidder 1 acquires the toehold. Let us see when this is the case.

Suppose that the target is selected at random from a large pool of companies. Then, each company is expected to be acquired with probability zero, so the price of a toehold is \( T(\theta, p) = \theta \times 1 = \theta \). Under such condition, it is optimal for bidder 1 to approach the target.

A more interesting case if when the target knows that it will be acquired at some point. First, let us calculate the expected stock price at the acquisition date. The distribution of the winning bid is between \( v (1, 0) \) and \( b \theta v (1, 1) \) according to distribution
\[
F_1^\theta (b) = \Pr [b_1 \leq b, b_2 \leq b] = \Pr [b_1 \leq b] \times \Pr [b_2 \leq b] = (1 - p)^{\frac{1}{1-\theta}} \left( \frac{v (1, 1) - v (1, 0)}{v (1, 1) - b} \right)^{2-\theta}.
\]

The corresponding probability density function is
\[
f_1^\theta (b) = (2 - \theta) (1 - p)^{\frac{2-\theta}{1-\theta}} (v (1, 1) - v (1, 0))^{2-\theta} \frac{1}{(v (1, 1) - b)^{3-\theta}}.
\]

The expected stock price at the acquisition date is then
\[
E [b_w] = v (1, 0) + (2 - \theta) (1 - p)^{\frac{2-\theta}{1-\theta}} (v (1, 1) - v (1, 0))^{2-\theta} \left[ \int_{v(1,0)}^{\theta b} \frac{b - v (1, 0)}{(v (1, 1) - b)^{3-\theta}} db \right] v(1,0)
\]
\[
= v (1, 0) + (2 - \theta) (1 - p)^{\frac{2-\theta}{1-\theta}} (v (1, 1) - v (1, 0))^{2-\theta} \left[ (2 - \theta) (1 - \theta) (v (1, 1) - b)^{2-\theta} \right] v(1,0)
\]
\[
= v (1, 0) + (v (1, 1) - v (1, 0)) \left[ 1 - (1 - p)^{\frac{1}{1-\theta}} \right] \left( 1 + \frac{p}{1-\theta} \right).
\]

Note that in the special case of \( \theta = 0 \), as before,
\[
E [b_w] = v (1, 0) + (v (1, 1) - v (1, 0)) [1 - (1 - p) (1 + p)]
\]
\[
= v (1, 0) + p^2 (v (1, 1) - v (1, 0)).
\]
It is easy to check that the expected revenues of the target are increasing in $\theta$:

$$
\frac{\partial \mathbb{E} [b_w]}{\partial \theta} = - (v(1,1) - v(1,0)) \left[ (1 - p) \frac{1}{1-\theta} \left( 1 + \frac{p}{1-\theta} \right) \right]'
= \frac{v(1,1) - v(1,0)}{(1-\theta)^2} (1-p) \frac{1}{1-\theta} \left[ (1 + \frac{p}{1-\theta}) \log \left( \frac{1}{1-p} \right) - p \right]
\geq \frac{v(1,1) - v(1,0)}{(1-\theta)^2} (1-p) \frac{1}{1-\theta} \left[ (1 + p) \log \left( \frac{1}{1-p} \right) - p \right]
> \frac{v(1,1) - v(1,0)}{(1-\theta)^2} (1-p) \frac{1}{1-\theta} [1 + p - p] > 0.
$$

References


