Financing Through Asset Sales*

Alex Edmans  
Wharton, NBER, and ECGI

William Mann  
Wharton

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Abstract

Most existing research on a firm’s financing decisions studies the choice between debt and equity issuance. This paper analyzes an alternative source of financing – selling non-core assets. We identify two new factors that drive a firm’s choice between financing through asset sales and equity issuance. First, investors in an equity issue share in the cash raised. Since the value of cash is certain, this mitigates the information asymmetry surrounding equity. Thus, in contrast to Myers and Majluf (1984), even if equity is more informationally-sensitive than non-core assets, the firm issues equity if the financing need is sufficiently high. The choice of financing depends on the amount required – low (high) financing needs are met by asset sales (equity issuance). Second, selling equity implies a low valuation not only for the equity being issued (the “lemons” problem) but also for the rest of the firm, since its value is perfectly correlated with the issued equity. In contrast, even if an asset seller suffers a “lemons” discount for the disposed asset, this need not imply a low valuation for the rest of the firm as the asset may be negatively correlated.

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*aedmans@wharton.upenn.edu, wmann@wharton.upenn.edu. We thank Gustavo Manso and seminar participants at Wharton for helpful comments. AE gratefully acknowledges financial support from the Dorinda and Mark Winkelman Distinguished Scholar award.
One of the most important decisions that a firm faces is how to raise financing. Most existing research focuses on the choice between debt and equity financing, with various theories identifying different factors that drive a firm’s security issuance decision. The trade-off theory argues that managers compare the benefits of debt (tax shields and a reduction in the agency costs of equity) against its costs (bankruptcy costs and the agency costs of debt). The pecking-order theory of Myers (1984), motivated by the model of Myers and Majluf (1984, “MM”), posits that managers issue securities that exhibit least information asymmetry. The market timing theory of Baker and Wurgler (2002) suggests that managers sell securities that are most overpriced by the market.

While there is substantial research on financing through security issuance, another major source of financing is relatively unexplored – selling non-core assets such as a division or a plant. Asset sales are substantial in reality: in 2010, Securities Data Corporation (SDC) records $133 billion in asset sales in the US1, compared to $130 billion in seasoned equity issuance. Although some of these sales may have been motivated by business reasons, capital raising is indeed an important motive for many disposals. Major firms in the oil and gas industry (including Chevron, Shell, and Conoco) have recently sold non-core divisions to raise capital for liquidity and debt service. Most notably, in October 2011, BP set a target of $45 billion in asset sales to cover the costs of the Deepwater Horizon spill. Banks worldwide have raised billions of dollars through asset sales in the recent crisis to reassure investors, replenish depleted capital, and build capital in anticipation of new regulatory standards. In September 2011, Banque Nationale de Paris and Société Générale announced plans to raise $96 billion and $5.4 billion respectively through asset sales, to create a financial buffer against possible contagion from other French banks. Bank of America raised $3.6 billion in August 2011 by selling a stake in a Chinese construction bank, and $755 million in November 2011 from disposing its stake in Pizza Hut. More broadly, the survey of Campello, Graham, and Harvey (2010) finds that 70% of financially constrained firms increased their asset sales in the financial crisis, compared to 37% of unconstrained firms. This difference also points to asset sales being used as a financing tool.

This paper analyzes the role of asset sales as a means of financing. In particular, it studies the conditions under which asset sales are preferable to equity issuance and vice-versa. We build a deliberately parsimonious model to maximize tractability; this allows for the key expressions to be solved for in closed form, and the economic forces driving the results to be transparent. The firm comprises a core asset and a non-core

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1This figure is an underestimate as 60% of asset sales in SDC have missing transaction values.
asset. The firm must raise financing to meet a liquidity need, and can do so either by issuing equity, which is a claim on the entire firm, or by selling part or all of the non-core asset. (If the non-core asset is indivisible, a partial asset sale reflects a carve-out.) The firm can be of two types; the value of the core asset is higher for the high type than the low type. The value of the non-core asset depends on how we specify the correlation between the core and non-core assets. When asset values are positively (negatively) correlated, the value of the non-core asset is higher (lower) for the high type.

It may seem that asset sales can already be analyzed by applying the general principles of the MM model of security issuance to the sale of assets, removing the need for a new theory specific to asset sales. Such an extension would suggest that assets are preferred to equity if they exhibit less information asymmetry, which occurs if the value of the non-core asset is less volatile than the core asset. While information asymmetry is indeed an important consideration, our model identifies several new distinctions between asset sales and equity issuance that also drive the financing choice, and may swamp information sensitivity considerations.

First, an advantage of equity issuance is that new shareholders obtain a stake in the entire firm. This includes not just the core and non-core assets in place (whose value is uncertain), but also the cash paid for the new shares. Since the value of this cash is certain, this mitigates the information asymmetry associated with the assets in place: the certainty effect. The certainty effect applies regardless of whether the new cash is held on the balance sheet or is used to repay debt, pay dividends, or finance an uncertain investment whose value is uncorrelated with firm type. In contrast, the purchaser of a non-core asset does not share in the value of the cash raised, and thus bears the full information asymmetry associated with the asset’s value. Hence, in contrast to MM, even if the non-core asset exhibits less information asymmetry than the firm as a whole, the manager may sell equity if enough cash is raised that the certainty effect outweighs the higher information asymmetry of equity. Formally, for low (high) financing requirements, the only equilibrium is a pooling equilibrium where both firm types sell assets (equity). Both pooling equilibria are sustainable for intermediate financing needs.

Since the certainty effect strengthens as the amount of financing increases, the model delivers the new result that the firm’s financing choice depends on the amount of financing required. This dependence contrasts standard financing models, where the choice depends only on inherent characteristics of the security (such as its level
of information asymmetry (MM) or misvaluation (Baker and Wurgler (2002)), and not the level of financing required – unless one assumes exogenous frictions such as transactions costs or limits on the amount of financing that can be raised through a given channel (e.g., notions of debt capacity). In our model, the level of financing influences the choice of financing, even though the entire financing need can be met through either source.

A second difference between asset sales and equity issuance is that the asset being sold need not be a carbon copy of the rest of the firm, and this represents an advantage to selling assets. When a firm issues equity, it suffers an Akerlof (1970) “lemons” discount on the equity issued – the market infers that the equity issued is lowly-valued, from the firm’s decision to issue it. This leads to the market not only paying a low price for the new equity issued, but also attaching a low valuation to the remainder of the firm. This is because the new equity being issued and the equity of the firm as a whole are necessarily perfectly correlated, since the former is a carbon copy of the latter. In contrast, when a firm sells non-core assets, it may receive a low price, but critically this need not imply a low valuation for the rest of the firm as a whole if the two are negatively correlated. We label this the correlation effect.

In the case of a negative correlation, three equilibria are possible. The first is a separating equilibrium where the high type sells assets and the low type issues equity. Even though the high type receives a (fair) low price for the assets he sells, so there are no market timing motives, this low price for assets does not imply a low valuation for the rest of the firm – indeed, he is inferred as a good type by selling assets and receives a high stock price. The low type receives a low stock price, but a deviation to asset sales would require selling his highly-valued assets at a low price instead. If his concern for the stock price (compared to fundamental value) is sufficiently small, the capital loss from deviation is more important than the stock price increase, and so the equilibrium is sustained. Thus, firms can sell poorly-performing assets without suffering negative inferences on the company as a whole. Note that no separating equilibrium is possible in the case of positive correlation, because the value of the claim being sold is necessarily positively correlated with the market value of the firm. The low type would always mimic the high type, receiving both a higher stock price and a capital gain on the high type’s financing method.

Second, there is a pooling equilibrium where both types sell assets. This equilibrium is relatively easy to sustain, as a deviation to equity issuance leads not only to a low price for the equity being sold, but also a low valuation on the rest of the firm.
The third is a pooling equilibrium where both types sell equity. This equilibrium is harder to sustain due to the correlation effect. If the manager’s stock price concerns are sufficiently weak, the only off-equilibrium path belief that satisfies the intuitive criterion is that an asset seller is the high type. Thus, even though he receives a low price for the assets being sold, as the market correctly infers that the assets are lowly valued from the manager’s decision to sell them, this does not imply a low valuation for the rest of the firm – indeed the market ascribes a high valuation. Thus, deviation is attractive, and so the equilibrium is unsustainable.

In sum, with negative correlation, and when the managers’ concern for the stock price is weak, the separating equilibrium is sustainable. When the stock price concern is high, at least one pooling equilibrium is sustainable. The parameter values under which the equity-pooling equilibrium is sustainable form a strict subset of those under which the asset-pooling equilibrium is sustainable – i.e., asset sales are easier to sustain than equity issuance. Thus, the correlation effect manifests itself in two ways: the sustainability of the separating equilibrium (which is not possible in a model of positive correlation), and the asset-pooling equilibrium being sustainable over a greater range of parameters than the equity-pooling equilibrium. Note that both the certainty and correlation effects apply to debt in the same way as equity: since debt is also a claim to the entire firm, it also benefits from the certainty effect and is positively correlated with the value of the firm. Thus, our model abstracts from debt financing and focuses on the choice between asset sales and equity issuance.

We extend the model to allow the cash to be used to finance an uncertain investment, whose expected value is correlated with firm type. It may appear that this extension should weaken the certainty effect, since the funds raised are now being used for uncertain investment rather than held as certain cash. This intuition turns out to be incomplete, because there is a second effect. The investment will be undertaken only if it is positive-NPV, and a positive-NPV investment increases the value of the firm’s balance sheet, to which equity investors (but not asset purchasers) have a claim. Put differently, while investors do not know the firm type, they do know that the funds they provide will increase in value, regardless of type. Thus, equity investors now have a claim to a larger certain value, and so the certainty effect strengthens. If the desirability of investment (for both firm types) is sufficiently high compared to the additional investment return generated by the high type over the low type, the second effect dominates – somewhat surprisingly, the certainty effect can strengthen when cash is used to finance an uncertain investment. Thus, equity issuance becomes
easier to sustain compared to the core model. In contrast, if investment becomes more volatile, the first effect is stronger and asset sales become preferable. Thus, the source of financing depends on the use of financing, even though we have a model of pure adverse selection with neither moral hazard nor bankruptcy costs. In all scenarios, for the case of positive correlation, it remains the case that asset (equity) sales are used for low (high) financing needs.

In ongoing work, we are extending the model to allow for positive and negative synergies between the core and non-core assets, and to allow firms to opt out of financing. These extensions highlight another advantage of asset sales, related to their operational motives. If the firm’s financing needs are private knowledge, the decision to issue equity is a negative signal as it implies that the firm’s cash position is weak (e.g., Miller and Rock (1985)). Asset sales have the advantage that they can be disguised as being motivated by business reasons (negative synergies) rather than financing reasons, and so they are greeted by a lower fall in market value.

Most of the existing literature on asset sales is empirical. Lang, Poulsen, and Stulz (1995) study why asset sales are typically associated with positive market reactions. They test two hypotheses – business reasons (asset sales lead to more efficient deployment of the assets) and financing reasons (asset sales are cheaper than alternative sources of financing) – and find support for the latter. Our paper provides a theoretical framework to explain why asset sales may indeed be a cheap source of financing. Slovin, Sushka, and Ferraro (1995) similarly find a positive market reaction to asset sales, and compare it to the market reaction to carve-outs and spin-offs. Maksimovic and Phillips (2001) analyze business reasons for asset sales – transferring the asset to a more productive owner – rather than financing motives. Bates (2006) studies the firm’s choice of how to use the proceeds from an asset sale, rather than what motivated the firm to sell assets in the first place.

Existing theoretical models consider asset sales as the only source of financing and do not compare it to equity issuance. Shleifer and Vishny (1992) show that it may be optimal to sell assets to an industry outsider rather than an industry rival. DeMarzo (2005) studies the conditions under which it is optimal to pool assets together rather than selling them individually. He (2009) generalizes the Leland and Pyle (1977) model to the case of multiple assets, and shows that the optimal amount to sell of each asset depends on their correlation. Milbradt (2012) and Bond and Leitner (2011) show that selling an asset will affect the market price of the seller’s remaining portfolio under mark-to-market accounting, changing his balance sheet constraint. We show that such
correlation effects are stronger for equity issuance: while a partial asset sale may imply a negative valuation of the remaining unsold non-core assets, it need not imply a negative valuation of the whole firm.

Since a partial asset sale in our model can also be interpreted as a carve-out, our paper is also related to the literature on carve-outs. Nanda (1991) also points out that non-core assets may be uncorrelated with the core business and that this may motivate a firm to issue equity at the subsidiary level. In his model, non-core assets always have a zero correlation, and the information asymmetry of core and non-core assets is the same. Our model allows for general correlations and information asymmetries, thus enabling us to generate the certainty and correlation effects. In addition, the manager in our model cares about the stock price as well as fundamental value, which allows for a richer set of equilibria including pooling equilibria; only separating equilibria exist in Nanda (1991). Empirically, Allen and McConnell (1998) study how the market reaction to carve-outs depends on the use of proceeds. Schipper and Smith (1986) show that equity issuance leads to negative abnormal returns, but carve-outs lead to positive returns.

The remainder of the paper is organized as follows. Section 1 outlines the general model. Section 2 analyzes the case of positive correlation and demonstrates the certainty effect. Section 3 studies negative correlation and introduces the correlation effect. In Section 4, the funds raised are used to finance an uncertain investment, and Section 5 concludes. Appendix A contains all proofs not in the main text and additional comparative statics; Appendix B contains other peripheral material.

1 The Model

The model consists of two types of agents: firms, which raise financing, and investors, who provide financing and set prices. We consider the financing division of an individual firm, which is one of two types: \( \theta = H \) or \( \theta = L \). The uninformed market places probability \( \pi \geq \frac{1}{2} \) on \( \theta = H \). The variable \( \pi \) captures the level of information asymmetry, with higher \( \pi \) corresponding to lower information asymmetry. Each firm is run by a manager who makes the financing decision. If the firm is of type \( \theta \), we say that the manager is also of type \( \theta \).

Each firm comprises two assets. The core business has value \( C_\theta \), where \( C_H > C_L \). The non-core business has value \( A_\theta \), and be considered either a physical asset (e.g. a plant) or a financial asset (e.g. an investment in another firm). Where there is no
ambiguity, we will use the term “assets” to refer to the non-core business. We consider two versions of the model. The first case is $A_H > A_L$, so that the value of the two assets is positively correlated. The second case is $A_L > A_H$, so the assets are negatively correlated. In both cases, we assume that:

\[ C_H + A_H > C_L + A_L, \]

i.e. type $H$ has a higher total value. Thus, even if assets are negatively correlated, the higher value of $L$’s non-core assets is outweighed by the lower value of its core assets. In Myers (1984), the key driver of financing choice is the volatility of the security being issued. The distinction between the two cases of $A_H > A_L$ and $A_H < A_L$ shows that it is not only the volatility of the non-core asset that matters ($|A_H - A_L|$), but also its correlation with the core asset ($\text{sign}(A_H - A_L)$).

We consider the financing problem of an individual firm. The firm must raise financing of $F < \min(A_L, A_H)$. The cash raised remains on the firm’s balance sheet. This can be interpreted as meeting a liquidity need (e.g. providing a buffer against future cash requirements) or a regulatory capital requirement (e.g. replenishing the balance sheet in response to an adverse shock). It also nests decisions to repay debt, pay dividends, or finance an uncertain investment whose value is uncorrelated with firm type – any policy that increases the firm’s value to equityholders by an amount $F$ in expectation, as long as the increase is uncorrelated with firm type and thus exhibits no information asymmetry. In Section 4 we allow the cash to fund an investment whose value is correlated with firm type, so that there is information asymmetry between managers and investors on the value of the investment.

We currently treat the firm’s financing need $F$ as exogenous. In MM, the firm has the option not to raise financing and instead to forgo investment; the goal of that paper is to show that information asymmetry can deter investment by hindering financing. Our focus, instead, is to study the choice between asset sales and equity to meet a given financing need, and so we take $F$ as given. In ongoing work we extend the model to allow for the firm’s financing needs to be privately known, so that it has a choice of whether to raise financing, and for positive and negative synergies between the core and non-core assets so there may be operational in addition to financing motives for asset sales.

The firm can raise funds either by selling assets or by issuing equity (which is a

\[ \text{We do not consider the case of } A_H = A_L \text{ as the non-core asset is now risk-free and it is automatic that the firm will always raise financing by selling it (as shown by MM).} \]
claim on both core and non-core assets); it cannot sell a claim to the core business alone as these assets are essential for the firm. (In the conclusion, we discuss an extension in which we relax this assumption.) Since $F < \min(A_L, A_H)$, it is possible to raise the required financing entirely through either source. We restrict attention to equilibria in which the firm is assumed to raise financing from a single source. This can be motivated by transactions costs associated with using multiple financing sources. (Appendix B formally derives conditions under which firms will not wish to deviate to multiple financing sources.)

We assume that the non-core asset is perfectly divisible so partial asset sales are possible; in the case of indivisible assets, a partial asset sale represents an equity carve-out. Formally, a firm of type $\theta$ issues a claim $K_\theta \in \{E, A\}$, where $K_\theta = E$ represents equity issuance and $K_\theta = A$ an asset sale. Investors infer the firm’s type based on its choice of claim $K_\theta$. These inferences affect both the firm’s current market valuation (also referred to as its stock price) and the terms at which it raises financing. Investors are perfectly competitive and price the claim being sold at its expected value, given the inferred firm type, so that they earn zero expected return. Thus, the firm may enjoy either an increase or decrease in its fundamental value, depending on whether it issues the claim at a gain or a loss. The manager’s objective function places weight $\omega$ on the firm’s stock price and $1 - \omega$ on fundamental value.

We use the Perfect Bayesian Equilibrium solution concept, which involves the following: (i) Investors have a belief about which manager types issue which claim $K_\theta$. (ii) The price of the claim being issued equals its expected value, conditional on investors’ beliefs in (i). (iii) Each manager type chooses to issue the claim $K_\theta$ that maximizes his objective function, given investors’ beliefs. (iv) Investors’ beliefs satisfy Bayes’ rule. Beliefs on claims $K_\theta$ issued off the equilibrium path satisfy the Cho and Kreps (1987) Intuitive Criterion (IC).

In general, there may be two types of equilibria: pooling equilibria, where both types issue the same claim ($K_H = K_L$) and separating equilibria, where each type issues a different claim ($K_H \neq K_L$). If a manager is indifferent between playing the equilibrium strategy and deviating, we assume that he remains with the equilibrium strategy. We first analyze the positive correlation version of the model ($A_H > A_L$) and then move to the negative correlation version ($A_L > A_H$).
2 Positive Correlation

We assume that $\omega = 0$ in this section for ease of exposition. The role of $\omega > 0$ only exists under negative correlation, as this creates a trade-off to being inferred as type $L$: market valuation falls, but the firm receives a high price if it sells assets. With a positive correlation, there is no such trade-off: being inferred as type $L$ is detrimental for both market valuation and fundamental value, and so $\omega > 0$ does not affect the sustainability of any equilibria.

With positive correlation, it is clear that there will be no separating equilibria. The use of claim $K_L$ immediately reveals the firm to be of type $L$. By deviating to issue claim $K_H$, firm type $L$ enjoys an increase in both its market value (as it is inferred as a high type) and fundamental value (as it receives a high price for the claim issued). Thus all of the equilibria will be pooling equilibria.

We start by deriving the conditions under which an asset-pooling equilibrium (APE) exists, and then move to the equity-pooling equilibrium (EPE).

2.1 Pooling Equilibrium, All Firms Sell Assets

We consider a pooling equilibrium in which all firms sell assets, supported by the off-equilibrium belief that anyone who sells equity is of type $L$. In this equilibrium, assets are valued at

$$E [A] = \pi A_H + (1 - \pi) A_L. \tag{2}$$

If equity is sold (off the equilibrium path), it is valued at

$$C_L + A_L + F.$$

The additional $F$ term arises because the cash that the firm receives from financing enters its balance sheet, and so new shareholders own a claim to this cash in addition to the two existing assets.$^3$

The fundamental value of type $L$ is thus given by:

$$C_L + A_L + F - \frac{FA_L}{E [A]} = C_L + A_L + \frac{\pi F(A_H - A_L)}{E [A]},$$

$^3$This is consistent with the treatment of financing in MM, although it plays no role in their analysis since both equity and debt are claims on the entire firm, which includes the financing raised.
and the fundamental value of type $H$ is:

$$C_H + A_H - \frac{(1 - \pi)F(A_H - A_L)}{E[A]}.$$  

(3)

Type $L$ enjoys a capital gain of \frac{\pi F(A_H - A_L)}{\pi A_H + (1 - \pi)A_L} by selling lowly-valued assets at a pooled price and so it is clear that he has no incentive to deviate. Type $H$ suffers a capital loss of \frac{(1 - \pi)F(A_H - A_L)}{\pi A_H + (1 - \pi)A_L} by selling highly-valued assets at a pooled price. He may thus deviate and issue equity. If he does so, fundamental value becomes

$$C_H + A_H - \frac{F(C_H + A_H - C_L - A_L)}{C_L + A_L + F}.$$  

(4)

The no-deviation (“ND”) condition is that (4) \leq (3), i.e.,

$$F \leq F^{A P E, N D} = \frac{E[A]}{A_H - E[A]}(C_H + A_H) - \frac{A_H}{A_H - E[A]}(C_L + A_L).$$  

(5)

This condition is equivalent to the “unit cost of financing” being lower for asset sales, i.e.

$$\frac{A_H}{E[A]} \leq \frac{C_H + A_H + F}{C_L + A_L + F}$$

where the numerator on each side is the true value of the claim being sold, and the denominator is the price that investors pay for that claim.

There are two forces that determine $H$’s incentives to deviate. The first is whether equity or assets are more information-sensitive. Since equity is a claim on both core assets and non-core assets, equity is relatively more information-sensitive the greater is \frac{C_H}{C_L} compared to \frac{A_H}{A_L}. This effect is a natural extension of the MM principle that high-type firms wish to issue claims that are least informationally-insensitive. Indeed, if \frac{A_H}{E[A]} > \frac{C_H + A_H}{C_L + A_L}, i.e. assets are sufficiently more volatile than equity, then the RHS of (5) is negative and so the pooling equilibrium is unsustainable for any $F$. Type $H$ then suffers a smaller capital loss by issuing undervalued equity compared to selling undervalued assets, and so will deviate to equity issuance.

The second force is the amount of financing $F$ being raised. This is unique to a model of asset sales and stems from the fact that the cash raised from financing enters into the firm’s balance sheet. Thus, if the investor purchases an equity claim, she shares in the value of this cash; but if she buys non-core assets, she does not. Since the value of cash is certain, this effect mitigates the information asymmetry associated with equity financing. This effect does not exist in MM, since only claims to the entire
firm (debt and equity) are considered, and so all claims share in the cash added to the balance sheet. Thus, the level of financing raised does not matter. Here, since equity investors but not asset purchasers obtain a claim to the cash raised, it has a differential effect on equity issuance and asset sales. Thus, there is an upper bound on $F$ to prevent deviation. If $F$ exceeds this upper bound, the certainty value of cash is sufficiently high that type $H$ decides to deviate to issuing equity – even though he is inferred as type $L$ by doing so. In particular, note that even if $\frac{A_H}{A_L} < \frac{C_H}{C_L}$, i.e. assets are less volatile than equity, it may be that (5) is violated so type $H$ issues equity. Thus, the MM result that the high type will issue the least volatile claim does not hold.

We now study the comparative statics on the upper bound (5). These are as follows:

$$\frac{\partial F^{APE,ND}}{\partial C_H} = \frac{E[A]}{(1 - \pi)(A_H - A_L)} > 0,$$

$$\frac{\partial F^{APE,ND}}{\partial (-C_L)} = \frac{A_H}{(1 - \pi)(A_H - A_L)} = 1 + \frac{\partial F^{APE,ND}}{\partial C_H} > 0,$$

$$\frac{\partial F^{APE,ND}}{\partial A_H} = \left(\frac{\pi}{1 - \pi}\right) - \frac{1}{1 - \pi} \left(\frac{A_L(C_H - C_L)}{(A_H - A_L)^2}\right) \leq 0,$$

$$\frac{\partial F^{APE,ND}}{\partial (-A_L)} = -\left(\frac{1}{1 - \pi}\right) - \frac{1}{1 - \pi} \left(\frac{A_H(C_H - C_L)}{(A_H - A_L)^2}\right) < 0,$$

$$\frac{\partial F^{APE,ND}}{\partial \pi} = \frac{1}{(1 - \pi)^2} \left(\frac{A_H}{A_H - A_L}\right)(C_H - C_L + A_H - A_L) > 0.$$

For the values of the non-core business, $C_H$ and $C_L$, the signs of the derivatives are intuitive: as we increase dispersion in the value of the core business ($C_H - C_L$), we increase the loss that $H$ makes by deviating to issue equity. This discourages deviation and the upper bound can relax, i.e., increase. The derivative with respect to $-C_L$ is larger because changes in $C_L$ have two effects. First, reducing $C_L$ increases the capital loss from equity issuance and so deters $H$ from selling equity. This effect is shared with increasing $C_H$: increasing $C_H$ and reducing $C_L$ both augment $C_H - C_L$. The second effect is specific to $C_L$. Reducing $C_L$ means that $H$ receives a lower price from deviating to sell equity, since equity is valued at $C_L + A_L$. This means that he has to sell a greater fraction of the firm’s equity, and so he bears the capital loss over a greater base. This second effect is shown by the additional 1 term in the $\frac{\partial F^{APE,ND}}{\partial (-C_L)}$ expression, and means that reducing $C_L$ dissuades type $H$ from issuing equity even more than increasing $C_H$.

Turning to the values of the non-core asset, the negative sign of $\frac{\partial F^{APE,ND}}{\partial (-A_L)}$ arises because lowering $A_L$ increases type $H$’s loss to selling assets for a pooled price, and thus encourages him to deviate to equity. This requires the upper bound on $F$ to
tighten, i.e. decrease. However, the sign of $\frac{\partial F_{APE,ND}}{\partial A_H}$ is ambiguous. There are two effects of increasing $A_H$. First, it increases the loss that $H$ makes from selling assets; this effect increases the incentive to deviate and is shared with reducing $A_L$. Second, it increases the pooled price that $H$ receives from selling assets (which is given by $E[A] = \pi A_H + (1 - \pi) A_L$), and so reduces the quantity of assets that the firm needs to sell. Thus, $H$ suffers the loss over a lower base, reducing the incentive to deviate. In contrast, reducing $A_L$ lowers the pooled price that $H$ receives from selling assets, increasing the quantity of assets that he needs to sell and causing a greater loss. Hence, both effects work in the same direction and there is no ambiguity with $A_L$.

Finally, the bound is increasing in $\pi$. As type $H$ begins to dominate the market, he suffers a lower capital loss from pooling on asset sales, because the pooled price $E[A]$ that he receives becomes closer to the true asset value of $A_H$. Thus, he has a lower incentive to deviate to selling equity.

We now verify whether the off-equilibrium belief, that an equity issuer is of type $L$, satisfies the IC. This is the case if type $L$ would weakly prefer to issue equity for some market inference; since he clearly will not deviate if revealed $L$, we check his action if revealed $H$. If equity issuers are inferred as type $H$, $L$’s fundamental value from deviation is:

$$C_L + A_L + F - F \left( \frac{C_L + A_L + F}{C_H + A_H + F} \right).$$

Thus, $L$ will weakly prefer to deviate, satisfying the IC, if:

$$F \leq F_{APE,IC} = \frac{A_L(C_H + A_H) - E[A](C_L + A_L)}{E[A] - A_L}. \quad (6)$$

It may seem that the IC is trivially satisfied since $L$ receives a high price for selling equity and being inferred as good, rather than a pooled price for selling assets. However, if $F$ is sufficiently large, selling equity is less attractive since the certainty effect reduces the gains from being inferred as a high type. Thus, we have another upper bound on $F$, again due to the certainty effect. If $\frac{E[A]}{A_L} > \frac{C_H + A_H}{C_L + A_L}$, i.e. assets are relatively more volatile than equity, then the RHS of (6) is negative and so the pooling equilibrium is unsustainable for any $F$. Type $L$ enjoys such a large capital gain from pooling on assets that he will not deviate to selling equity even if revealed good.\(^4\)

Lemma 1 below summarizes the equilibrium. The proof shows that the IC condition

\(^4\)To eliminate an equilibrium with $F > F_{APE,IC}$ via the IC, we also require that type $H$ will deviate if he is revealed good. This will automatically be the case, as he will break even rather than suffering a capital loss. In all of the other equilibria that we consider, it will similarly be automatic that type $H$ will deviate if he is revealed good, so we will not need to show this mathematically.
is stronger than the ND condition, and so the former is necessary and sufficient for the pooling equilibrium to hold.

Lemma 1. (Positive correlation, pooling equilibrium, all firms sell assets.) A pooling equilibrium is sustainable in which all firms sell assets ($K_H = K_L = A$) and a firm that issues equity is inferred as type $L$, if

$$F \leq F^{APE,IC} = \frac{A_L(C_H + A_H) - E[A](C_L + A_L)}{E[A] - A_L}. $$

The upper bound $F^{APE,IC}$ is increasing in $C_H$ and $-C_L$ and decreasing in $A_H$, $-A_L$, and $\pi$.

2.2 Pooling Equilibrium, All Firms Sell Equity

We now consider the alternative pooling equilibrium in which both types issue equity, supported by the off-equilibrium belief that anyone who sells assets is of type $L$. Equity is valued at

$$E[C + A] + F = \pi (C_H + A_H) + (1 - \pi) (C_L + A_L) + F$$

and if assets are issued (off the equilibrium path), they are valued at $A_L$.

The fundamental value of type $L$ is

$$C_L + A_L + F \left(\frac{\pi(C_H - C_L + A_H - A_L)}{E[C + A] + F}\right),$$

and the fundamental value of type $H$ is

$$C_H + A_H - F \left(\frac{(1 - \pi)(C_H - C_L + A_H - A_L)}{E[C + A] + F}\right).$$

As in the previous pooling equilibrium, type $L$ makes a capital gain and it is automatic that he will not deviate. Type $H$ will not deviate to selling assets if:

$$F \geq F^{EPE,ND} = \frac{A_L(C_H + A_H) - A_H E[C + A]}{A_H - A_L}. \quad (7)$$

In contrast to Section 2.2, the ND condition now imposes a lower bound on $F$. This also results from the certainty effect. If $F$ is sufficiently high, type $H$ suffers a relatively small loss from selling equity at a pooled price, since the undervaluation is mitigated.
by the certainty effect, and so will not deviate. Note that if \( \frac{A_H}{A_L} > \frac{C_H + A_H}{E(C + A)} \), and so assets are more volatile than equity, the RHS of (7) is negative. Thus, regardless of the level of \( F \), the inequality is satisfied and \( H \) will not deviate from equity issuance to asset sales. The comparative statics on (7) are given in the Appendix.

We next verify whether the off-equilibrium belief, that an asset seller is of type \( L \), satisfies the IC. If \( L \) sells assets and is inferred as type \( H \), his fundamental value becomes:

\[
C_L + A_L + F - F \left( \frac{A_L}{A_H} \right),
\]

and so \( L \) will weakly prefer to deviate (satisfying the IC) if

\[
F \geq F_{EPE, IC} = \frac{A_L E[C + A] - A_H (C_L + A_L)}{A_H - A_L}.
\] (8)

Again, it is not trivial that the IC is satisfied: \( F \) must be sufficiently high that the certainty effect reduces the capital gain from pooling on equity, so that \( L \) prefers to deviate and sell assets.

Lemma 2 below summarizes the equilibrium. The proof shows that the IC condition is stronger than the ND condition, and so the former is necessary and sufficient for the pooling equilibrium to be sustained.

**Lemma 2.** (Positive correlation, pooling equilibrium, all firms sell equity.) A pooling equilibrium is sustainable in which all firms sell assets (\( K_H = K_L = A \)) and a firm that sells assets is inferred as type \( L \), if

\[
F \geq F_{EPE, IC} = \frac{A_L E[C + A] - A_H (C_L + A_L)}{A_H - A_L}.
\]

The lower bound \( F_{EPE, IC} \) is increasing in \( C_H \), \(-C_L \) and \( \pi \), and decreasing in \( A_H \) and \(-A_L \).

### 2.3 Comparing the Equilibria

We now analyze the conditions under which each equilibrium is sustainable. The results are given in Proposition 1 below:

**Proposition 1.** (Positive correlation, comparison of equilibria.) An asset-pooling equilibrium is sustainable if \( F \leq F_{APE, IC} \) and an equity-pooling equilibrium is sustainable if \( F \geq F_{EPE, IC} \), where \( F_{APE, IC} \) and \( F_{EPE, IC} \) are given by (6) and (8) respectively and \( F_{APE, IC} > F_{EPE, IC} \). Thus, if:
Proposition 1 shows that, when the amount of financing required increases, firms switch from selling assets to issuing equity, since the magnitude of the certainty effect is increasing in the magnitude of financing raised. Thus, the amount of financing required determines the type of financing chosen – firms only issue equity to raise large amounts of financing, even though there are no fixed costs of equity issuance. In the standard pecking order, the type of security issued only depends on the security’s inherent characteristics, such as its information asymmetry or overvaluation, unless one assumes exogenous limitations on financing such as limited debt capacity. Here, there are no limits as the amount of financing required can be fully raised by either source.

3 Negative Correlation

Under the case of negative correlation, there are now potentially two reasons for type \( H \) to prefer asset sales. First, selling equity may reveal him to be of type \( L \) and lead to a low stock price, as in the case of positive correlation. Second, he may now make a capital gain from selling assets, since the firm’s assets are worth only \( A_H \), the lower of the possible values. The latter effect was not present in the case of positive correlation. Since the two effects may work in opposite directions, we now return to general \( \omega \), so that the manager cares about the stock price as well as fundamental value.

We start by deriving the conditions under which a separating equilibrium \((SE)\) exists, then turn to the two pooling equilibria \(APE\) and \(EPE\).

3.1 Separating Equilibrium

We consider a separating equilibrium in which type \( H \) sells assets and type \( L \) issues equity. Thus, the value of assets sold is \( A_H \), and the value of equity sold is \( C_L + A_L \). Since both firms are issuing claims that are fairly valued, the fundamental value of each firm is not affected by the financing. Thus, the fundamental and market values of type \( \theta \) are given by \( C_\theta + A_\theta \).\(^5\)

\(^5\)Another potential separating equilibrium involves type \( H \) issuing equity and type \( L \) selling assets. Under such an equilibrium, \( H \) receives a fair (high) price for the equity he issues and \( L \) receives a
It is clear that type $H$ will not deviate as his stock price will fall to $C_L + A_L$, and his fundamental value will fall as he will be issuing underpriced equity rather than selling a fairly-priced asset. Type $H$ suffers a correct, low valuation on the asset that he sells, which is correctly assessed by the market as being a “lemon”, and so the “market timing” motive for financing (e.g. Baker and Wurgler (2002)) does not exist. However, under negative correlation, the low valuation on the assets sold does not imply a low valuation for the rest of the firm. Thus, $H$ is willing to sell assets despite receiving a low price for them.

If type $L$ deviates, his fundamental value will fall from $C_L + A_L$ to $C_L + A_L + F(A_H - A_L)$. Crucially, the third term is negative, since $A_L > A_H$: $L$ suffers a capital loss, which offsets the fact that his market value rises from being inferred as type $H$. Thus, if:

$$\omega \leq \omega^{SE} = \frac{F(A_L - A_H)}{A_H} + (C_H - C_L) - (A_L - A_H),$$

then $L$ will not deviate. Equation (9) requires the manager’s weight on the stock price $\omega$ to be low, so that he prefers to suffer a low market value from being revealed as type $L$, rather than a capital loss from deviating to sell assets. From (1), the RHS is less than 1 so this is not trivially satisfied.

The sustainability of $SE$ stems from the negative correlation between core and non-core assets, which is at the heart of this paper. Type $H$ can separate himself by choosing a claim that is not a carbon copy of (is negatively correlated with) the rest of the firm. Note that this correlation effect is absent in a standard financing model of security issuance. Since debt and equity are both positively correlated with the value of the firm, there is no way to achieve a separating equilibrium in such models if the firm’s financing need is exogenous. The “fundamental” and “market value” motives would work in the same direction: type $L$ would always mimic type $H$, as he would enjoy a capital gain and be inferred as a good type.

The results of this subsection are summarized in Lemma 3 below:
Lemma 3. (Negative correlation, separating equilibrium.) A separating equilibrium is sustainable in which type $H$ sells assets and type $L$ sells equity ($K_H = A$, $K_L = E$), if

$$\omega \leq \omega^{SE} = \frac{F(A_L - A_H)}{A_H F(A_L - A_H) + (C_H - C_L) - (A_L - A_H)}.$$  

The upper bound $\omega^{SE}$ is decreasing in $C_H$ and $-C_L$, and increasing in $A_H$, $-A_L$, and $F$.

The upper bound $\omega^{SE}$ is increasing in $F$ – but the role of $F$ is different from in the pooling equilibria of Section 2. The certainty effect is not relevant in $SE$, as here equity is issued at a fair price, rather than a pooled price. Instead, the role of $F$ is as follows: if type $L$ deviates to selling assets, he makes a capital loss. If $F$ is high, he suffers this loss on a large base, and so is less willing to deviate. The intuition behind the other comparative statics is given in the Appendix.

3.2 Pooling Equilibrium, All Firms Sell Assets

As in Section 2.1, we consider a pooling equilibrium in which all firms sell assets, supported by the off-equilibrium belief that anyone who sells equity is of type $L$. As before, sold assets are valued at $E[A] = \pi A_H + (1 - \pi) A_L$ and issued equity is valued at $C_L + A_L + F$.

It is automatic that $H$ will not deviate, as he is making a capital gain from selling lowly-valued assets at a pooled price. By pooling, $L$’s objective function is

$$\omega E[C + A] + (1 - \omega) \left( C_L + A_L + F - F \frac{A_L}{E[A]} \right).$$

He gains by being valued at a pooled stock price, and loses by selling assets at a pooled price. If $L$ deviates to issuing equity, he issues a fairly-priced claim and so makes no capital gain nor loss; similarly, the firm is valued correctly. Thus, his fundamental and market values will be $C_L + A_L$. He will not deviate if:

$$\omega \geq \omega^{APE,ND} = \frac{F \left( \frac{A_L - A_H}{E[A]} \right)}{(C_H - C_L) - (A_L - A_H) + F \left( \frac{A_L - A_H}{E[A]} \right)}.$$

We now have a lower bound on $\omega$ to prevent deviation. If the manager’s stock price concerns $\omega$ are sufficiently low, he will deviate to sell equity – even though he suffers
a low market value, this is outweighed by the fact that he no longer makes a capital loss. From (1), the RHS is less than 1, so the inequality can always be satisfied.

The lower bound given by (10) is relatively loose, i.e., easy to satisfy. It is relatively easy to rule out a deviation to equity issuance. Issuing equity not only leads to a low price (of $C_L + A_L$) on the equity being sold (as in MM), but also implies a low valuation (of $C_L + A_L$) for the rest of the firm. This is because the correlation between the equity being sold and the rest of the firm is necessarily 1. The second effect is absent in MM, since the manager only cares about fundamental value and not the stock price. As we will see in the equity-pooling equilibrium, this will not be the case when considering deviations to asset sales, since the asset being sold is not a carbon copy of the rest of the firm.

Finally, it is automatic that the off-equilibrium-path belief (that a firm that sells equity is of type $L$) satisfies the IC. Type $L$ will indeed deviate to equity if revealed $H$: his stock price will rise, and he will receive a capital gain by selling equity for a high price, compared to his current loss for selling highly-valued assets at a pooled price.

The results of this subsection are summarized in Lemma 4 below:

**Lemma 4.** *(Negative correlation, pooling equilibrium, all firms sell assets.)* A pooling equilibrium is sustainable in which both types sell assets ($K_H = K_L = A$) and a firm that sells equity is inferred as type $L$, if

$$\omega \geq \omega^{APE,ND} = \frac{F \left( \frac{A_L - A_H}{E[A]} \right)}{((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L - A_H}{E[A]} \right)}.$$  

The lower bound $\omega^{APE,ND}$ is increasing in $F$, $\pi$, and $A_L - A_H$, and decreasing in $C_H - C_L$.

The bound is increasing in $F$: higher $F$ increases the capital loss that $L$ suffers from pooling, as a greater quantity of the undervalued assets needs to be sold. This makes pooling harder to sustain. Put differently, if $F$ is high, $L$ suffers such a large capital loss from selling assets that it prefers to “bite the bullet” and issue equity even though this leads to a low market valuation. The intuition behind the other comparative statics is given in the Appendix.
3.3 Pooling Equilibrium, All Firms Sell Equity

We finally consider a pooling equilibrium in which all firms sell equity, supported by the off-equilibrium belief that anyone who sells assets is of type $L$. As before, issued equity is valued at $E[C + A] + F$ and sold assets are valued at $A_L$.

It is automatic that $L$ will not deviate, as he is selling lowly-valued equity at a pooled price. Type $H$, however, is suffering a capital loss. By pooling, his objective function is:

$$\omega E[C + A] + (1 - \omega) \left( C_H + A_H + F - F \frac{C_H + A_H + F}{E[C + A] + F} \right).$$

If he deviates to selling assets, he enjoys a capital gain but suffers a lower stock price. His objective function becomes:

$$\omega (C_L + A_L) + (1 - \omega) \left( C_H + A_H + F - F \frac{A_H}{A_L} \right).$$

Thus, we again obtain a lower bound on $\omega$ to prevent deviation:

$$\omega \geq \omega_{EPE,ND} = \frac{F \left( \frac{C_H + A_H + F}{E[C + A] + F} - \frac{A_H}{A_L} \right)}{\pi \left( (C_H - C_L) - (A_L - A_H) \right) + F \left( \frac{C_H + A_H + F}{E[C + A] + F} - \frac{A_H}{A_L} \right)}.$$ (11)

The comparative statics are given in the Appendix.

Unlike in Section 3.2, it is not automatic that the off-equilibrium belief satisfies the IC. If $L$ deviates to asset sales and is revealed $H$, his payoff becomes

$$\omega (C_H + A_H) + (1 - \omega) \left( C_L + A_L + F - F \frac{A_L}{A_H} \right),$$

whereas his current payoff is

$$\omega E[C + A] + (1 - \omega) \left( C_L + A_L + F - F \frac{C_L + A_L + F}{E[C + A] + F} \right).$$

Thus, he will deviate if:

$$\omega \geq \omega_{EPE,IC} = \frac{F \left( \frac{A_L}{A_H} - \frac{C_L + A_L + F}{E[C + A] + F} \right)}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L}{A_H} - \frac{C_L + A_L + F}{E[C + A] + F} \right)}.$$ (12)
This is a lower bound, and is strictly between 0 and 1. If $L$ deviates to asset sales and is inferred as type $H$, he enjoys an increase in market value but suffers a fall in fundamental value, because he will sell highly-valued assets at a loss. Thus, only if the manager’s stock price concerns are sufficiently high will he deviate. Note that the IC was trivially satisfied in $APE$ where the deviation involved issuing equity — if the deviator is inferred as type $H$, not only does this lead to a high market valuation, but also a high valuation of the equity being issued: both the equity being sold, and the rest of the firm, are valued at the same price $(C_H + A_H)$ since the former is a carbon copy of the latter. Here, the deviation is to assets, which are not a carbon copy of the firm and so can be priced differently: even though the deviator enjoys a high market valuation (of $C_H + A_H$), he suffers a loss on the assets being sold (which fetch only $A_H$).

Lemma 5 below summarizes the equilibrium. The proof shows that the IC condition is stronger than the ND condition, and so the former is necessary and sufficient for the pooling equilibrium to be sustained.

**Lemma 5.** (Negative correlation, pooling equilibrium, all firms sell equity.) A pooling equilibrium is sustainable in which all firms sell assets ($K_H = K_L = A$) and a firm that sells assets is inferred as type $L$, if

$$\omega \geq \omega^{EPE, IC} = \frac{F \left( \frac{A_L}{A_H} - \frac{C_L + A_L + F}{E[C + A] + F} \right)}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L}{A_H} - \frac{C_L + A_L + F}{E[C + A] + F} \right)}.$$

The lower bound $\omega^{EPE, IC}$ is increasing in $\pi$, $A_H$ and $-A_L$. The effect of $C_H$ and $C_L$ is ambiguous.

### 3.4 Comparing the Equilibria

We now analyze the circumstances under which each equilibrium is sustainable. The results are given in Proposition 2 below:

**Proposition 2.** (Negative correlation, comparison of equilibria.) A separating equilibrium is sustainable if $\omega \leq \omega^{SE}$, an asset-pooling equilibrium is sustainable if $\omega \geq \omega^{APE, ND}$, and an equity-pooling equilibrium is sustainable if $\omega > \omega^{EPE, IC}$, where $\omega^{SE}$, $\omega^{APE, ND}$, and $\omega^{EPE, IC}$ are given by (9), (10), and (12), respectively and $\omega^{APE, ND} < \omega^{SE} < \omega^{EPE, IC}$. Thus, if:

(i) $0 < \omega < \omega^{APE, ND}$, only the separating equilibrium is sustainable,
both the separating and asset-pooling equilibria are sustainable,

(iii) $\omega^{SE} < \omega < \omega^{EPE,IC}$, only the asset-pooling equilibrium is sustainable,

(iv) $\omega^{EPE,IC} \leq \omega < 1$, both the asset-pooling and equity-pooling equilibria are sustainable.

The thresholds $\omega^{APE,ND}$, $\omega^{SE}$, and $\omega^{EPE,IC}$ are all increasing in $F$.

Proposition 2 shows that, for the case of negative correlation, asset sales will be more commonly used for financing than equity issuance. This can be seen in two ways. First, for any $\omega$, there is at least one sustainable equilibrium (separating or pooling) in which type $H$ sells assets, but only for the highest values of $\omega$ ($\omega \geq \omega^{EPE,IC}$) will he sell equity. Second, the range of $\omega$’s over which $EPE$ is sustainable is a strict subset of the range of $\omega$’s over which $APE$ is sustainable. Thus, for $\omega^{APE,ND} \leq \omega \leq \omega^{SE}$, $APE$ is sustainable, but $EPE$ is not.

The intuition behind the preference for asset sales is as follows. In both equilibria, for the IC to be satisfied, it must be that type $L$ is willing to deviate if it will be inferred as type $H$. For $APE$, this deviation involves selling equity. Since the equity being sold necessarily has a correlation of 1 with the firm’s market value, it is automatic that $L$ will deviate as it will obtain a not only a high market valuation for the rest of the firm, but also a high price for the equity being sold. Thus, the only constraint is the ND condition, which is also relatively easy to satisfy. In contrast, for $EPE$, $L$’s deviation involves selling assets. Even if deviation leads to $L$ receiving a high market value, it will lead to a low price on the sold assets due to the negative correlation. If the second force is stronger, condition (12) is violated, and so the only permissible off-equilibrium belief is that an asset seller is of type $H$. This belief does satisfy the IC as $H$ will now receive a high market value rather than a pooled market value, and will also break even on the asset sale rather than making a loss on equity issuance. Thus, $H$ will deviate to sell assets and $EPE$ is unsustainable. Intuitively, $H$ is willing to sell assets, even though the market correctly infers that they are “lemons” – because the low valuation of the sold assets need not imply a low valuation for the rest of the firm, due to the negative correlation. This intuition did not apply when we considered a deviation to equity issuance from $APE$, because such a deviation leads not only to a “lemons” discount on the equity being sold, but also a low valuation for the rest of the firm.

In sum, the correlation effect – and its implications for the desirability of financing through asset sales – manifests in two ways. First, a separating equilibrium is sus-
tainable, whereas it was unattainable in the positive-correlation model. Even though
type H receives a low price from the assets that it sells, this does not imply a low
valuation for the rest of the firm and so H is willing to sell assets. Second, the APE
is sustainable over a greater range of parameters than EPE.

This analysis points to an interesting benefit of diversification. Stein (1997) noted
that an advantage of holding assets that are not perfectly correlated is that a conglomer-
erate can engage in “winner-picking”, i.e. increase investment in the division that has
the best investment opportunities at the time. Our model suggests that an advantage
of diversification is “loser-picking”: a firm can sell a lowly-valued asset, thus raising
financing, without implying a low value for the rest of the firm. Non-core assets may
thus be seen as a form of financial slack, or financing capacity: they can be sold (even
if they turn out to be poorly-performing) without adversely affecting the value of the
rest of the firm. Indeed, they may be preferable to debt as a source of financing as they
need not be positively correlated with the rest of the firm. In contrast, since debt and
equity are typically positively correlated, the issuance of debt may imply that debt is
overvalued and thus the remainder of firm is also overvalued.

Note that the preference for asset sales exists even though assets may be more
informationally sensitive than core equity. If \( \frac{A_L}{A_H} > \frac{C_H + A_H}{C_L + A_L} \) (which is fully consistent
with the conditions \( C_H + A_H > C_L + A_L \) and \( A_L > A_H \)), then assets are more volatile
and the MM principle would suggest that equity issuance should be preferred. In
contrast, we show that asset sales are may be preferred due to the correlation effect.

Finally, all the bounds are increasing in the amount of financing \( F \), but \( F \) plays a
different role here than in the positive correlation model, where it was important due to
the certainty effect. Here, a greater \( F \) means that a capital gain or loss from deviating
is sustained over a larger base, and thus becomes more important relative to the stock
price change from deviating. It effectively increases the fundamental value motive
relative to the market value motive, and requires a higher weight on the market value
\( \omega \) to maintain indifference. This effect was absent from the positive correlation section,
as there was no trade-off between the stock price and fundamental value motives.
Increasing \( F \) makes the separating equilibrium easier to sustain (L is less willing to
deviate and receive the high stock price as it will suffer a greater capital loss), but
both pooling equilibria harder to sustain (deviation will lead to a greater improvement
in fundamental value). As in the core model, the choice of financing depends on the
amount of financing required, but the source of its importance is different.


4 Cash Used For Investment

In the core model, the cash $F$ was used for activities with a certain value: either it remained on the balance sheet, or it was paid out to firm claimants. This section extends the model to allow for the cash to be used to finance investment with an uncertain value.

We first note that, since all agents are risk-neutral, only expected values matter. Thus, the model is unchanged if we simply make the investment opportunity uncertain, so that its payoff is a random variable with expected value equal to $F$ for both types. For the investment opportunity to affect the results, it must vary with firm type so that there is information asymmetry between the manager and investors regarding the value of the investment. We thus assume that $F$ is used to finance an investment which pays expected value $R_\theta$ in firm type $\theta$ (again, only expected values matter). We parameterize $R_\theta = F(1 + r_\theta)$ where:

\[
\frac{r_H}{1 + r_L} \geq \frac{C_H + A_H}{C_L + A_L}.
\]

(13)

The first assumption implies that the investment is better for type $H$, and positive-NPV for both firm types. If the investment were not positive-NPV, the manager would not undertake it. The second assumption is a technical condition that means that investment is not too volatile. This assumption is useful for a clear ordering of the IC and ND conditions for the positive correlation case: as in the core model, the IC condition is stronger, and so it is sufficient for the equilibrium to be sustainable.

Intuitively, it would seem that using cash for an uncertain investment will weaken the certainty effect and make asset sales more desirable than equity issuance, but we will show that this is not necessarily the case. We start by studying positive correlation and then move to negative correlation.

4.1 Positive Correlation

We first consider the pooling equilibrium where all firms sell assets and an equity issuer is inferred as type $L$. As before, assets are valued at $E[A] = \pi A_H + (1 - \pi)A_L$, and issued equity is now valued at $C_L + A_L + F(1 + r_L)$. The fundamental value of type $\theta$ is given by:

\[
C_\theta + A_\theta + F(1 + r_\theta) - F\frac{A_\theta}{E[A]}.
\]
As before, $L$ has no incentive to deviate, but $H$ is making a capital loss and so may deviate and issue equity. If he does so, his fundamental value becomes

$$C_H + A_H + F(1 + r_H) - F \left( \frac{C_H + A_H + F(1 + r_H)}{C_L + A_L + F(1 + r_L)} \right).$$

As is intuitive, $C_\theta$ and $R_\theta (= F(1 + r_\theta))$ enter symmetrically in all expressions, since the funds raised are invested. A purchaser of equity receives a share of $C$, $R$ and $A$, but a purchaser of assets receives only a share of $A$. The uncertainty of the investment thus increases the information sensitivity of core equity.

Type $H$ will not deviate if:

$$F [A_H (1 + r_L) - E[A] (1 + r_H)] \leq E[A] (C_H + A_H) - A_H (C_L + A_L). \quad (15)$$

As in the core model (see equation (5)), if $E[A] (C_H + A_H) < A_H (C_L + A_L)$, then the ND condition can never be satisfied and $APE$ is unsustainable. We thus focus on the case in which $E[A] (C_H + A_H) \geq A_H (C_L + A_L)$. We first assume that $\frac{A_H}{E[A]} > \frac{1}{1 + r_L}$, i.e. investment is not too volatile. The LHS of (15) is positive, and so we again have an upper bound on $F$, given by:

$$F \leq \frac{E[A] (C_H + A_H) - A_H (C_L + A_L)}{A_H (1 + r_L) - E[A] (1 + r_H)}. \quad (16)$$

In the core model (equation (5)), the denominator is $A_H - E[A]$, which is what we obtain by setting $r_L = r_H = 0$. Thus, if $\frac{r_H}{r_L} < \frac{A_H}{E[A]}$, the denominator is greater if cash is used for investment, and so the bound is tighter: it is harder to support $APE$. This result may appear surprising, since it seems that for any $r_H > r_L$, the certainty effect becomes weaker since the cash is now used for uncertain investment. Put differently, the volatility in the investment $R$ effectively increases the volatility in the core asset (recall that $C$ and $R$ enter symmetrically), and the latter makes equity issuance less attractive and $APE$ easier to sustain.

The above intuition is incomplete, since using cash to finance investment has two effects. They can be best seen by the following decomposition of the investment returns:

$$R_L = F(1 + r_L)$$

$$R_H = F(1 + r_L) + F(r_H - r_L).$$

The first, intuitive effect is the $F(r_H - r_L)$ term which appears in the $R_H$ equation
only: using cash for an uncertain investment weakens the certainty effect. The value of the investment is greater for the high type and so it suffers a greater capital loss from selling equity. However, there is a second effect, captured by the $F(1 + r_L)$ term which is common to both types. This increases the certainty effect: since the investment is positive-NPV, it means that an equity investor now has a claim to a larger certain value: $F(1 + r_L)$ rather than $F$. Due to this second effect, $r_H > r_L$ is not sufficient for the upper bound to be increased and $APE$ to be easier to sustain. If $\frac{r_H}{r_L} < \frac{A_H}{A_L}$, then the difference in returns is not sufficient to outweigh the first effect, and it is harder to prevent type $H$ from deviating. Only if $\frac{r_H}{r_L} > \frac{A_H}{A_L}$, i.e. investment is highly volatile, then the LHS of (15) is non-positive and so the ND condition is always satisfied: $APE$ is sustainable for any $F$.

The IC condition is satisfied if:

$$F(E[A](1 + r_L) - A_L(1 + r_H)) \leq A_L(C_H + A_H) - E[A](C_L + A_L)$$

The contrast with the core model (equation (6)) is similar for the ND conditions. If $\frac{1+r_H}{1+r_L} \geq \frac{E[A]}{A_L}$, the LHS of equation (17) is non-positive and so the IC condition is satisfied for all $F$. If instead $\frac{1+r_H}{1+r_L} < \frac{E[A]}{A_L} < \frac{r_H}{r_L}$, the upper bound on $F$ becomes looser than in the case where the cash remains on the balance sheet, and the IC condition is easier to satisfy. The volatility of the investment increases $L$’s incentives to deviate and be revealed as $H$, since he will receive a capital gain on the investment value $R$ in addition to the core asset value $C$, neither of which he receives by pooling on asset sales. However, if $\frac{E[A]}{A_L} > \frac{r_H}{r_L}$, then the bound becomes tighter: since the investment is positive-NPV, it increases the certainty effect and thus reduces the desirability of issuing equity, even if $L$ is revealed as type $H$.

Lemma 6 below summarizes the equilibrium. The proof shows that the IC condition is stronger than the ND condition, and so the former is necessary and sufficient for the pooling equilibrium to be sustained.

**Lemma 6.** (Positive correlation, pooling equilibrium, all firms sell assets, cash used for investment.) A pooling equilibrium is sustainable in which all firms sell assets ($K_H = K_L = A$) and a firm that sells equity is inferred as type $L$, if

$$F(E[A](1 + r_L) - A_L(1 + r_H)) \leq A_L(C_H + A_H) - E[A](C_L + A_L).$$

Compared to the case where cash remains on the balance sheet (Lemma 1):
(i) If \( \frac{r_H}{r_L} < \frac{E[A]}{A_L} \), the upper bound on \( F \) is tighter and the asset-pooling equilibrium is sustainable across a smaller range of \( F \),

(ii) If \( \frac{1+r_H}{1+r_L} < \frac{E[A]}{A_L} < \frac{r_H}{r_L} \), the upper bound on \( F \) is looser and the asset-pooling equilibrium is sustainable across a larger range of \( F \),

(iii) If \( \frac{1+r_H}{1+r_L} \geq \frac{E[A]}{A_L} \), the asset-pooling equilibrium is sustainable for all \( F \).

We now turn to the equity-pooling equilibrium. The effect of using cash for uncertain investment is similar to \( APE \). Intuitively, it may seem that this usage will always make \( EPE \) harder to satisfy (i.e., raise the lower bound on \( F \)) because the volatility of the investment reduces the certainty effect. However, if \( r_H \) is sufficiently close to \( r_L \), this volatility effect is outweighed by the fact that the investment is positive-NPV and so increases the certain amount to which equity investors have a claim from \( F \) to \( F(1 + r_H) \). Thus, the lower bound on \( F \) loosens and the equilibrium becomes easier to satisfy. Since the economics are similar, we move immediately to the statement of the equilibrium in Lemma 7 below and defer the full analysis to the proofs.

**Lemma 7.** (Positive correlation, pooling equilibrium, all firms sell equity, cash used for investment.) A pooling equilibrium is sustainable in which all firms sell equity \((K_H = K_L = E)\) and a firm that sells assets is inferred as type \( L \), if

\[
F(A_H(1 + r_L) - A_L(1 + E[r_0])) \geq A_L E[C + A] - A_H(C_L + A_L).
\]

where \( E[r_0] = \pi r_H + (1 - \pi) r_L \). Compared to the case where cash remains on the balance sheet (Lemma 2):

(i) If \( \frac{E[r_0]}{r_L} < \frac{E[A]}{A_L} \), the lower bound on \( F \) is looser and the equity-pooling equilibrium is sustainable across a greater range of \( F \)

(ii) If \( \frac{1+E[r_0]}{1+r_L} < \frac{E[A]}{A_L} < \frac{E[r_0]}{r_L} \), the lower bound on \( F \) is tighter and the equity-pooling equilibrium is sustainable across a smaller range of \( F \)

(iii) If \( \frac{1+E[r_0]}{1+r_L} \geq \frac{A_H}{A_L} \), the equity-pooling equilibrium is unsustainable for all \( F \).

The comparison of equilibria is summarized in Proposition 3:

**Proposition 3.** (Positive correlation, cash used for investment, comparison of equilibria.) An asset-pooling equilibrium is sustainable if \( F \leq F^{APE, IC, I} \), and an equity-pooling equilibrium is sustainable if \( F \geq F^{EPE, IC, I} \), where \( F^{APE, IC, I} \) and \( F^{EPE, IC, I} \) are given
by:

\[ F^{APE,IC,I} = \begin{cases} \frac{A_L(C_H + A_H) - E[A](C_L + A_L)}{E[A](1 + r_L) - A_L(1 + r_H)} & \text{for } \frac{1 + r_H}{1 + r_L} < \frac{E[A]}{A_L}, \\ \infty & \text{for } \frac{1 + r_H}{1 + r_L} > \frac{E[A]}{A_L} \end{cases} \]

\[ F^{EPE,IC,I} = \begin{cases} \frac{A_L E[C_A] - A_H (C_L + A_L)}{A_H (1 + r_L) - A_L (1 + E[r_H])} & \text{for } \frac{1 + E[r_H]}{1 + r_L} < \frac{A_H}{A_L}, \\ \infty & \text{for } \frac{1 + E[r_H]}{1 + r_L} > \frac{A_H}{A_L} \end{cases} \]

and \( F^{APE,IC,I} \geq F^{EPE,IC,I} \). Thus, if:

(i) \( F \leq F^{EPE,IC,I} \), only an asset-pooling equilibrium is sustainable,

(ii) \( F^{EPE,IC,I} < F < F^{APE,IC,I} \), both the asset-pooling and equity-pooling equilibria are sustainable,

(iii) \( F \geq F^{APE,IC,I} \), only an equity-pooling equilibrium is sustainable.

The thresholds \( F^{APE,IC,I} \) and \( F^{EPE,IC,I} \) are both increasing in \( r_H \) and decreasing in \( r_L \).

Proposition 3 demonstrates the robustness of the results of the core model to allowing cash to be used for a volatile investment rather than remaining on the balance sheet. Regardless of \( r_H \) and \( r_L \), it remains the case that \( APE \) is sustainable for low \( F \) and the \( EPE \) is sustainable for high \( F \): it is never the case that the volatility of the investment causes the certainty effect to “reverse” and mean that asset (equity)-pooling is now sustainable for high (low) \( F \). As in the core model, the source of financing depends on the amount of financing raised.

In addition to demonstrating the robustness of this idea from the core model, this extension also demonstrates a new result. As \( r_H \) rises and \( r_L \) falls, the upper bound on \( APE \) loosens and the lower-bound on the \( EPE \) tightens. Indeed, if \( r_H \) is sufficiently greater than \( r_L \), the bound becomes infinite: \( APE \) is sustainable for all \( F \) (since the upper bound is now infinity) and \( EPE \) is sustainable for no \( F \) (since the lower bound is now infinity). This can cause the equilibrium to shift from asset sales to equity sales.\[7\]

Thus, the source of financing also depends on the use of financing: if the funds raised will be used for volatile investments, it is more likely to be raised from asset sales rather than equity issuance. The source of financing can depend on the use of financing in models of moral hazard (uses that are more likely to be subject to agency problems will be financed by debt rather than equity, to avoid the agency costs of dispersed equity) or bankruptcy costs (purchases of tangible assets are more likely to financed by debt

\[7\] Formally, a given \( F \) under which both pooling equilibria were sustainable in the core model may now support only \( APE \), when cash is used for investment. A given \( F \) under which only the \( EPE \) was sustainable in the core model may now support both equilibria, or only \( APE \).
rather than equity), but here we deliver this dependence in a model of pure adverse selection, without moral hazard or bankruptcy costs. In addition, our predictions for the use of equity financing differ from a model of moral hazard. With moral hazard, if cash is to remain on the balance sheet for general corporate purposes (rather than to finance a specific investment), debt financing will be preferred to equity issuance to avoid the agency costs of free cash flow (Jensen (1986)). Here, equity financing is preferred due to the certainty effect.

4.2 Negative Correlation

We now move to the case of negative correlation, which turns out to be very similar to the core model. The separating equilibrium, where $H$ sells assets and $L$ issues equity, is unchanged. As before, $H$ has no incentive to deviate as he will suffer a capital loss on undervalued equity and a lower stock price. The ND condition for $L$ is unchanged from (9):

$$\omega \leq \omega^{SE} = \frac{F(A_L - A_H)}{A_H} + (C_H - C_L) - (A_L - A_H).$$

The new parameters for the investment return only matter when equity is misvalued, but this deviation condition involves either fairly-valued equity or undervalued assets.

Similarly, for $APE$, the ND condition for type $L$ is unchanged from (10):

$$\omega \geq \omega^{APE,ND} = \frac{F \left( \frac{A_L - A_H}{E[A]} \right)}{(C_H - C_L) - (A_L - A_H) + F \left( \frac{A_L - A_H}{E[A]} \right)}.$$

Again, the deviation condition involves either fairly-valued equity or undervalued assets, and so is unaffected by the return parameters. As in the core model, it is automatic that type $H$ will not deviate, and the intuitive criterion will be satisfied.

The equity-pooling equilibrium does change, and the results are given by Lemma 8 below:

**Lemma 8.** (Negative correlation, pooling equilibrium, all firms sell equity, cash used for investment.) A pooling equilibrium is sustainable in which all firms sell assets
\( (K_H = K_L = A) \) and a firm that sells assets is inferred as type \( H \), if

\[
\omega \geq \omega_{EPE,IC,I} = \frac{F \left( \frac{A_L}{A_H} - \frac{C_L + A_L + F (1 + r_L)}{E[C + A] + F (1 + E[r])] \right)}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L}{A_H} - \frac{C_L + A_L + F (1 + r_L)}{E[C + A] + F (1 + E[r])] \right)}
\]

where \( E[r_L] = \pi r_L + (1 - \pi)r_L \). Compared to the case where cash remains on the balance sheet (Lemma 5):

(i) If \( E[r_L] > E[C + A] + F \sqrt{C_L + A_L + F} \) the equity-pooling equilibrium is sustainable across a smaller range of \( \omega \);

(ii) If \( E[r_L] < E[C + A] + F \sqrt{C_L + A_L + F} \) the equity-pooling equilibrium is sustainable across a larger range of \( \omega \).

As in Lemma 5, there is a lower bound on \( \omega \) to ensure that type \( L \) will be willing to deviate to asset sales if he is revealed as type \( H \). Intuitively, it would seem that using cash for volatile investment would increase the lower bound and make the equilibrium harder to sustain: since type \( L \) is now making a capital gain on the investment \( R \) as well as the core asset \( C \), he is less willing to deviate from equity pooling, violating the IC. However, this intuition only holds if investment is sufficiently volatile, i.e. \( E[r_L] > E[C + A] + F \sqrt{C_L + A_L + F} \). If instead \( E[r_L] < E[C + A] + F \sqrt{C_L + A_L + F} \), this effect is outweighed by the second effect which also exists in the positive correlation section: since the investment is positive-NPV, an equity investor now has a claim to a larger certain value \( F (1 + r_L) \) rather than \( F \) which reduces the capital gain from issuing overvalued equity and encourages deviation. Thus, the IC condition is easier to satisfy, and the equilibrium is sustainable for a larger range of \( \omega \).

The comparison of equilibria is given by Proposition 4, and is analogous to Proposition 2.

**Proposition 4.** (Negative correlation, cash used for investment, comparison of equilibria.) A separating equilibrium is sustainable if \( \omega \leq \omega_{SE} \), an asset-pooling equilibrium is sustainable if \( \omega \geq \omega_{APE,ND} \), and an equity-pooling equilibrium is sustainable if \( \omega > \omega_{EPE,IC,I} \), where \( \omega_{SE} \), \( \omega_{APE,ND} \), and \( \omega_{EPE,IC,I} \) are given by (9), (10), and (18), respectively and \( \omega_{APE,ND} < \omega_{SE} < \omega_{EPE,IC,I} \). Thus, if:

(i) \( 0 < \omega < \omega_{APE,ND} \), only the separating equilibrium is sustainable,

(ii) \( \omega_{APE,ND} \leq \omega \leq \omega_{SE} \), both the separating and asset-pooling equilibria are sustainable,

(iii) \( \omega_{SE} < \omega < \omega_{EPE,IC,I} \), only the asset-pooling equilibrium is sustainable,
(iv) \( \omega^{EPE,IC} \leq \omega < 1 \), both the asset-pooling and equity-pooling equilibria are sustainable.

The thresholds \( \omega^{APE,ND} \), \( \omega^{SE} \), and \( \omega^{EPE,IC} \) are all increasing in \( F \).

5 Conclusion

This paper has studied a firm’s choice between raising financing through selling non-core assets, and issuing equity. One relevant consideration is the relative information sensitivity of non-core assets and equity value, a natural extension of the Myers and Majluf (1984) insight. This paper introduces two important additional effects that drive a firm’s financing decision. First, investors in an equity issue share in the value of the cash raised from the issue, but purchasers of non-core assets do not. Since the value of cash is certain, this mitigates the information asymmetry associated with issuing uncertain claims: the certainty effect. Thus, even if the firm’s equity has a more uncertain valuation than its non-core assets, an equity issue may be preferred to an asset sale (in contrast to the MM prediction) if the financing need is sufficiently high. A firm’s choice of financing thus depends on the level of financing required – in the case of positive correlation, low financing needs are met through asset sales and high financing needs are met through equity issuance, even though we have assumed no exogenous limitations on the amount of financing that can be raised through a particular method nor transactions costs. This result remains robust to allowing the cash to be used to finance an uncertain investment: asset (equity) sales are used for low (high) financing needs. Somewhat surprisingly, if cash is used to finance an uncertain investment, the certainty effect may strengthen: the asset-pooling equilibrium becomes easier to sustain and the equity-pooling equilibrium becomes harder to sustain.

Second, a disadvantage of equity issuance is that the market attaches not only a low valuation to the equity being sold, but also to the remainder of the firm, since the equity being sold and the remaining equity are necessarily perfectly correlated. This need not be the case for an asset sale, since the asset being sold is not a carbon copy of the remainder of the firm. Thus, even if the market correctly assesses the sold asset to be lowly valued, and so the firm suffers a “lemons” discount on the sold asset, this does not imply a low valuation for the rest of the firm: the correlation effect. This effect can lead to asset sales being strictly preferred to equity issuance.

In ongoing work, we are extending the analysis to allow for non-zero synergies between the core- and non-core businesses, and also for the firm’s financing needs to be
private knowledge: the firm has a choice of whether to raise financing. This extension aims to capture an additional advantage of financing via asset sales. Typically, the need to raise financing is a negative signal as it implies that the firm’s cash position is weaker than previously thought (e.g. Miller and Rock (1985)). However, if a firm raises financing by selling assets, it can disguise a financing need as one that is business-motivated, i.e. driven by the desire to dispose of a non-synergistic asset. Since the market does not know the motive for the asset sale, the reaction to raising financing via selling assets is less negative.

An additional extension allows the firm to sell a claim to the core asset alone. This extension demonstrates the robustness of the ideas of the core model. One of the assets (core or non-core) will be more informationally-sensitive than the other, and so the information sensitivity of equity will lie in between. It may therefore seem (from MM) that the sale of one asset will always dominate equity issuance, since one of the assets will have lower information sensitivity than equity. However, even though equity is more information-sensitive, it may still be preferred due to the certainty effect – indeed, in the core model, the asset-pooling equilibrium may be unsustainable, and the equity-pooling equilibrium may be sustainable, even if equity is more information-sensitive than assets. Allowing the firm to sell the core asset in the negative-correlation model of Section 3 also gives the firm a choice of the asset correlation (and so it is privately known), while in the current model it is nature that decides whether the asset is positively- or negatively-correlated (and so the correlation is publicly known). In the extension, the firm can either sell the core asset (which is positively-correlated with firm value) or the non-core asset (which is negatively-correlated). We will provide full analyses of these extensions in a future draft.
References


A Proofs and Additional Comparative Statics

Proof of Lemma 1

The IC condition (6) is stronger than the ND condition (5) if

$$\left[ \frac{E[A]}{E[A] - A_L} - \frac{A_H}{A_H - E[A]} \right] (C_L + A_L) < \left[ \frac{E[A]}{A_H - E[A]} - \frac{A_L}{E[A] - A_L} \right] (C_H + A_H)$$

This is true if and only if \( \pi \) is greater than

$$\pi^{APE,IC} = \frac{\sqrt{A_H A_L} - A_L}{A_H - A_L} < \frac{1}{2}$$

which is then satisfied because \( \pi \geq \frac{1}{2} \). Therefore, the IC condition (6) is sufficient for the ND condition (5) to be satisfied.

The comparative statics with respect to \( F^{APE,IC} \) are as follows:

\[
\begin{align*}
\frac{\partial F^{APE,IC}}{\partial C_H} &= \frac{A_L}{\pi(A_H - A_L)} > 0, \\
\frac{\partial F^{APE,IC}}{\partial (-C_L)} &= 1 + \frac{A_L}{\pi(A_H - A_L)} > 0, \\
\frac{\partial F^{APE,IC}}{\partial A_H} &= -\frac{A_L(C_H - C_L)}{\pi(A_H - A_L)^2} < 0, \\
\frac{\partial F^{APE,IC}}{\partial (-A_L)} &= -\frac{1 - \pi}{\pi} - \frac{A_H (C_H - C_L)}{\pi(A_H - A_L)^2} < 0, \\
\frac{\partial F^{APE,IC}}{\partial \pi} &= -\frac{1}{\pi^2} \left( \frac{A_L}{A_H - A_L} \right) (C_H - C_L + A_H - A_L) < 0.
\end{align*}
\]

The intuition behind these first four comparative statics is similar to for \( F^{APE,IC} \), except that the sign of \( \frac{\partial F^{APE,IC}}{\partial A_H} \) is now unambiguous. The sign of \( \frac{\partial F^{APE,IC}}{\partial \pi} \) is negative, whereas before we had \( \frac{\partial F^{APE,IC}_{\pi}}{\partial \pi} > 0 \). An increase in \( \pi \) raises the capital gain that \( L \) enjoys from selling assets at a pooled price, and so reduces his incentive to deviate to issuing equity.

Comparative Statics on Equation (7)

The comparative statics on (7), the ND bound for \( EPE \) in the positive correlation
case, are as follows:

\[
\frac{\partial F^{EPE,ND}}{\partial C_H} = (1 - \pi) \frac{A_H}{A_H - A_L} - 1, \\
\frac{\partial F^{EPE,ND}}{\partial (-C_L)} = (1 - \pi) \frac{A_H}{A_H - A_L} > 0, \\
\frac{\partial F^{EPE,ND}}{\partial A_H} = -(1 - \pi)(C_H - C_L) \frac{A_L}{(A_H - A_L)^2} - \pi < 0, \\
\frac{\partial F^{EPE,ND}}{\partial (-A_L)} = -(1 - \pi)(C_H - C_L) \frac{A_H}{(A_H - A_L)^2} < 0, \\
\frac{\partial F^{EPE,ND}}{\partial \pi} = -\left( A_L \left( \frac{C_H - C_L}{A_H - A_L} + 1 \right) + (C_H - C_L) + (A_H - A_L) \right) < 0.
\]

As with APE, the derivative on \(C_H\) is one less than the derivative on \(-C_L\), which makes the effect of increasing \(C_H\) ambiguous. There are two effects of changing \(C_H\) and \(C_L\). First, increasing \(C_H\) and reducing \(C_L\) augments the loss that \(H\) suffers from pooling, encouraging deviation and thus raising the lower bound on \(F\) for the equilibrium to be sustainable. Second, increasing \(C_H\) raises the (pooled) price that \(H\) gets from issuing equity. This reduces the fraction of equity he needs to sell, and thus decreases the loss, which acts in the opposite direction to the first effect. In contrast, decreasing \(C_L\) reinforces the first effect as it raises the fraction of equity he needs to sell and thus the loss.

Increasing dispersion in \(A\) augments the loss \(H\) suffers if he deviated to selling assets. Thus, the lower bound on \(F\) loosens (becomes smaller). The effect of \(A_H\) is greater than that of \(A_L\) because it increases the pooled price that \(H\) receives from selling equity, which reduces his capital loss. The intuition behind \(\frac{\partial F^{EPE,ND}}{\partial \pi} < 0\) is standard.

**Proof of Lemma 2**

Rewrite the ND bound (7) as

\[
\frac{(1 - \pi)A_L(C_H - C_L) + (1 - \pi)A_L(A_H - A_L) - (A_H - A_L)(E[C + A])}{A_H - A_L}.
\]
Thus, the IC condition (8) is sufficient for (7), i.e. $F_{EPE,IC} \geq F_{EPE,ND}$, if and only if

$$\pi C_H A_L + (1 - \pi) A_L (C_L + A_L - A_H) - C_L A_H$$

$$\geq (1 - \pi) A_L (C_H - C_L) + (1 - \pi) A_L (A_H - A_L)$$

$$- (A_H - A_L) \left[ \pi (C_H + A_H) + (1 - \pi) (C_L + A_L) \right].$$

Solving for $\pi$, this condition is equivalent to

$$\pi \geq \frac{A_L (C_H - C_L + A_H - A_L)}{(A_H + A_L) (C_H - C_L + A_H - A_L)} = \frac{A_L}{A_H + A_L}.$$

This lower bound is less than $\frac{1}{2}$ since $A_L < A_H$ under positive correlation. Since $\pi \geq \frac{1}{2}$, we indeed have $F_{EPE,IC} \geq F_{EPE,ND}$.

As in APE, the comparative statics are unambiguous for the IC condition, and the intuition is standard:

$$\frac{\partial F_{EPE,IC}}{\partial C_H} = \frac{\pi A_L}{A_H - A_L} > 0,$$

$$\frac{\partial F_{EPE,IC}}{\partial (-C_L)} = 1 + \frac{\pi A_L}{A_H - A_L} > 0,$$

$$\frac{\partial F_{EPE,IC}}{\partial A_H} = - \frac{\pi A_L (C_H - C_L)}{(A_H - A_L)^2} < 0,$$

$$\frac{\partial F_{EPE,IC}}{\partial (-A_L)} = (1 - \pi) - \frac{\pi A_H (C_H - C_L)}{(A_H - A_L)^2} < 0,$$

$$\frac{\partial F_{EPE,IC}}{\partial \pi} = \frac{C_H - C_L}{A_H - A_L} + 1 > 0.$$

**Proof of Proposition 1**

We only need to show that $F_{APE,IC} > F_{EPE,IC}$. The difference between the two bounds $F_{APE,IC} - F_{EPE,IC}$ is given by:

$$\left( \frac{1}{\pi} - \pi \right) \left( \frac{A_L}{A_H - A_L} \right) (C_H - C_L + A_H - A_L) > 0,$$

which is positive as required.

**Proof of Lemma 3**
The comparative statics on the upper bound $\omega^{SE}$ are as follows:

\[
\frac{\partial \omega^{SE}}{\partial C_H} = \frac{\partial \omega^{SE}}{\partial (-C_L)} = -\frac{F(A_L-A_H)}{A_H} < 0,
\]

\[
\frac{\partial \omega^{SE}}{A_L} = \left(\frac{F(C_H-C_L)}{A_H} \right)^2 > 0,
\]

\[
\frac{\partial \omega^{SE}}{\partial (-A_H)} = \left(\frac{F(A_L-A_H)}{A_H} \left(1 + \frac{F_A}{A_H}\right) \right) > 0,
\]

\[
\frac{\partial \omega^{SE}}{\partial F} = \left(\frac{F(A_L-A_H)}{A_H} + \left(\frac{(C_H-C_L) - (A_L-A_H)}{A_H}\right) \right)^2 > 0.
\]

Unlike in the positive correlation case, this derivative is symmetric with respect to both $C_H$ and $-C_L$. Increasing dispersion in $C$ raises $L$’s incentive to be inferred as type $H$ by deviating to asset sales, which forces the bound on $\omega$ to tighten. Increasing the dispersion of $A$ raises the capital loss that $L$ would suffer if he deviated to asset sales, so it is easier to achieve separation and the bound on $\omega$ can loosen. The intuition for the comparative statics with respect to $F$ are given in the core text.

**Proof of Lemma 4**
The comparative statics are as follows:

\[
\frac{\partial \omega_{APE,ND}}{\partial C_H} = \frac{\partial \omega_{APE,ND}}{\partial (-C_L)} = -F \left( \frac{A_L - A_H}{E[A]} \right) \frac{1}{\left( (C_H - C_L) - (A_L - A_H) + F \left( \frac{A_L - A_H}{E[A]} \right) \right)^2} < 0,
\]

\[
\frac{\partial \omega_{APE,ND}}{\partial A_L} = F \times \frac{A_L - A_H}{E[A]} + \left( (C_H - C_L) - (A_L - A_H) \right) \frac{A_H}{E[A]} \frac{1}{\left( (C_H - C_L) - (A_L - A_H) + F \left( \frac{A_L - A_H}{E[A]} \right) \right)^2} > 0,
\]

\[
\frac{\partial \omega_{APE,ND}}{\partial (-A_H)} = F \times \frac{A_L - A_H}{E[A]} + \left( (C_H - C_L) - (A_L - A_H) \right) \frac{A_H}{E[A]} \frac{1}{\left( (C_H - C_L) - (A_L - A_H) + F \left( \frac{A_L - A_H}{E[A]} \right) \right)^2} > 0,
\]

\[
\frac{\partial \omega_{APE,ND}}{\partial F} = \pi \left( (C_H - C_L) - (A_L - A_H) \right) \left( \frac{A_L}{E[A]} - 1 \right) \frac{1}{\left( (C_H - C_L) - (A_L - A_H) + F \left( \frac{A_L - A_H}{E[A]} \right) \right)^2} > 0,
\]

\[
\frac{\partial \omega_{APE,ND}}{\partial \pi} = \frac{F \left( (C_H - C_L) - (A_L - A_H) \right) \left( \frac{A_L - A_H}{E[A]} \right)^2}{\left( (C_H - C_L) - (A_L - A_H) + F \left( \frac{A_L - A_H}{E[A]} \right) \right)^2} > 0.
\]

The intuition for the comparative statics with respect to \( C_H, -C_L, A_H, \) and \( A_L \) is standard, and the intuition for \( F \) is given in the main text. An increase in \( \pi \) lowers the pooled asset price (since \( \pi \) is the weight on the low asset value \( A_H \)), augmenting \( L \)'s capital loss to pooling on assets and requiring a higher weight \( \omega \) on stock price to deter deviation. However, it also increases the pooled stock price and so \( L \)'s market value benefit from pooling, allowing a lower weight on \( \omega \) to deter deviation. The first effect turns out to be larger, so overall the bound is increasing in \( \pi \).

**Comparative Statics of Equation (11)**

For notational convenience, label the equity values:

\[
E_H \equiv C_H + A_H + F,
\]

\[
E_M \equiv \pi(C_H + A_H) + (1 - \pi)(C_L + A_L) + F,
\]

\[
E_L \equiv C_L + A_L + F.
\]
The comparative statics on the lower bound $\omega^{EPE,ND}$ are as follows:

$$\frac{\partial \omega^{EPE,ND}}{\partial C_H} = -\pi \omega^{EPE,ND} + F(1 - \pi)(1 - \omega^{EPE,ND}) \frac{E_L}{E_M} \frac{E_H}{E_M} \frac{A_H}{A_L}$$

$$= \pi F \times \left[ \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right) + \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right) \left( 1 - \pi \right) \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right) \left( 1 - \pi \right) \right] \leq 0,$$

$$\frac{\partial \omega^{EPE,ND}}{\partial (-C_L)} = -\pi \omega^{EPE,ND} + F(1 - \pi)(1 - \omega^{EPE,ND}) \frac{E_H}{E_M} \frac{A_H}{A_L}$$

$$= \pi F \times \left[ \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right) + \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right) \left( 1 - \pi \right) \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right) \left( 1 - \pi \right) \right] \leq 0,$$

$$\frac{\partial \omega^{EPE,ND}}{\partial A_L} = \frac{\omega^{EPE,ND} \pi - (1 - \omega^{EPE,ND}) F(1 - \pi) \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right)}{\pi((C_H - C_L) - (A_L - A_H)) + F \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right)}$$

$$= \pi F \times \left[ \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right) \left( 1 - \pi \right) \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right) \left( 1 - \pi \right) \right] \leq 0,$$

$$\frac{\partial \omega^{EPE,ND}}{\partial (-A_H)} = \frac{\omega^{EPE,ND} \pi + (1 - \omega^{EPE,ND}) \left( \frac{1}{A_L} \right) \left( 1 - \pi \right) F \frac{E_L}{E_M}}{\pi((C_H - C_L) - (A_L - A_H)) + F \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right)}$$

$$= \pi F \times \left[ \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right) + \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right) \left( 1 - \pi \right) \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right) \left( 1 - \pi \right) \right] > 0,$$

$$\frac{\partial \omega^{EPE,ND}}{\partial F} = \frac{(1 - \omega^{EPE,ND}) \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right)}{\pi((C_H - C_L) - (A_L - A_H)) + F \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right)}$$

$$= \pi \frac{\left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right) \left( 1 - \pi \right) \left( \frac{E_H}{E_M} - \frac{A_H}{A_L} \right) \left( 1 - \pi \right) \right] > 0.$$
The effect of $C_H$ on the lower bound $\omega^{EPE,ND}$ is ambiguous: an increase in $C_H$ raises the pooled stock price and thus increases $H$’s market value motive not to deviate (reducing the required $\omega$), but also increases $H$’s capital loss from pooling on equity sales (requiring a higher $\omega$). Thus, whether the manager’s objective should shift more to the stock price or to fundamental value to prevent deviation depends on which effect is stronger. The effect of $C_L$ is ambiguous for similar reasons.\(^8\)

The effect of $A_L$ is also ambiguous. Increasing $A_L$ raises the pooled stock price, reducing $H$’s incentive to deviate. It also affects $H$’s fundamental motive to deviate, but this effect is ambiguous. An increase in $A_L$ raises the capital gain from deviating to asset sales, augmenting the incentive to deviate. On the other hand, it reduces the capital loss from pooling on equity issuance, by raising the pooled stock price. This lowers the incentive to deviate.

The bound is increasing in $-A_H$. A fall in $A_H$ reduces $H$’s stock price under pooling and so lowers the loss in market value from deviation. Thus, his weight on market value must increase to maintain indifference. In addition, a fall in $A_H$ also increases the fundamental motive to deviate: it raises the capital gain from selling assets and the capital loss from issuing equity. Thus, a fall in the weighting on fundamental value is required to maintain indifference. Both effects require $\omega^{EPE,ND}$ to increase. The bound is also increasing in $F$. A higher $F$ causes $H$ to suffer the capital loss from pooling over a larger base, which increases his fundamental motive to deviate. Thus, a fall in his weight on fundamental value – an increase in $\omega$ – is needed to

\(^8\)More precisely, the lower bound $\omega^{EPE,ND}$ is increasing in $C_H$ if and only if

$$\pi \omega^{EPE,ND} < (1 - \omega^{EPE,ND}) \times (1 - \pi) \frac{F(C_L + A_L + F)}{(E[C + A] + F)^2},$$

where the LHS captures the stock price effect and the RHS captures the capital loss effect.

The lower bound $\omega^{EPE,ND}$ is increasing in $-C_L$ if and only if

$$\pi \omega^{EPE,ND} < (1 - \omega^{EPE,ND}) \times (1 - \pi) \frac{F(C_H + A_H + F)}{(E[C + A] + F)^2}.$$
maintain indifference. Finally, the bound is decreasing in $\pi$. High $\pi$ means that $H$ suffers a smaller fundamental loss to pooling on equity issuance, and also raises the pooled stock price, raising the market value motive not to deviate. Thus, the required weighting on the stock price need not be so high to encourage cooperation.

**Proof of Lemma 5**

We must verify that the IC condition (12) is stronger than the ND condition (11). This is true if and only if:

$$\pi \left( \frac{A_L}{A_H} - \frac{C_L + A_L + F}{E[C + A] + F} \right) > (1 - \pi) \left( \frac{C_H + A_H + F}{E[C + A] + F} - \frac{A_H}{A_L} \right)$$

$$\frac{\pi A_L^2 + (1 - \pi) A_H^2}{A_H A_L} > \frac{\pi(C_L + A_L) + (1 - \pi)(C_H + A_H) + F}{E[C + A] + F}$$  \hspace{1cm} (19)

When $\pi = \frac{1}{2}$, this collapses to

$$\frac{A_L}{A_H} - \frac{C_L + A_L + F}{E[C + A] + F} > \frac{C_H + A_H + F}{E[C + A] + F} - \frac{A_H}{A_L}$$

$$\iff \frac{A_L^2 + A_H^2}{A_H A_L} > \frac{C_H + A_H + C_L + A_L + 2F}{E[C + A] + F}$$

$$\iff \frac{A_H^2 + A_L^2}{2A_H A_L} > \frac{E[C + A] + F}{E[C + A] + F} = 1$$

$$\iff 0 < A_H^2 - 2A_H A_L + A_L^2 = (A_H - A_L)^2$$

Thus, the IC condition is stronger when $\pi = \frac{1}{2}$. Moreover, for general $\pi$, the LHS of (19) is increasing in $\pi$ while the RHS is decreasing in $\pi$. Thus, for any $\pi \geq \frac{1}{2}$, the IC condition is stronger.
Turning to the comparative statics, we have:

\[ \frac{\partial \omega^{EPE,IC}}{\partial C_H} = \frac{\pi(1 - \omega^{EPE,IC}) F \frac{E_L}{E_M} - (1 - \pi)\omega^{EPE,IC}}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L}{A_H} - \frac{E_L}{E_M} \right)} \]

\[ = (1 - \pi) \frac{\pi((C_H - C_L) - (A_L - A_H)) \frac{E_L}{E_M} - \left( \frac{A_L}{A_H} - \frac{E_L}{E_M} \right)}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L}{A_H} - \frac{E_L}{E_M} \right)} \leq 0, \]

\[ \frac{\partial \omega^{EPE,IC}}{\partial (-C_L)} = \frac{\pi(1 - \omega^{EPE,IC}) F \frac{E_H}{E_M} - (1 - \pi)\omega^{EPE,IC}}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L}{A_H} - \frac{E_L}{E_M} \right)} \]

\[ = (1 - \pi) \frac{\pi((C_H - C_L) - (A_L - A_H)) \frac{E_H}{E_M} - \left( \frac{A_L}{A_H} - \frac{E_L}{E_M} \right)}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L}{A_H} - \frac{E_L}{E_M} \right)} \leq 0, \]

\[ \frac{\partial \omega^{EPE,IC}}{\partial A_L} = \frac{\omega^{EPE,IC} (1 - \pi) - (1 - \omega^{EPE,IC}) F \left( \frac{\pi E_H}{E_M} - \frac{1}{A_H} \right)}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L}{A_H} - \frac{E_L}{E_M} \right)} \]

\[ = (1 - \pi) \left[ \frac{\pi E_H}{E_M} - \left( \frac{A_L}{A_H} - \frac{E_L}{E_M} \right) \right] + F \left( \frac{A_L}{A_H} - \frac{E_L}{E_M} \right) > 0, \]

\[ \frac{\partial \omega^{EPE,IC}}{\partial (-A_H)} = \frac{\omega^{EPE,IC} (1 - \pi) - (1 - \omega^{EPE,IC}) F \left( \frac{\pi E_L}{E_M} - \frac{A_L}{A_H} \right)}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L}{A_H} - \frac{E_L}{E_M} \right)} \]

\[ = (1 - \pi) \left[ \frac{\pi E_L}{E_M} - \left( \frac{A_L}{A_H} - \frac{E_L}{E_M} \right) \right] + F \left( \frac{A_L}{A_H} - \frac{E_L}{E_M} \right) > 0, \]

\[ \frac{\partial \omega^{EPE,IC}}{\partial F} = -\frac{(1 - \omega^{EPE,IC}) \left( \frac{E_L}{E_M} - \frac{A_L}{A_H} + F \left( \frac{E_M - E_L}{E_M} \right) \right)}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L}{A_H} - \frac{E_L}{E_M} \right)} \]

\[ = -\frac{(1 - \pi)((C_H - C_L) - (A_L - A_H)) \left( \frac{E_L}{E_M} + \left( \frac{F}{E_M} \right) \right) (1 - \frac{E_L}{E_M}) - \frac{A_L}{A_H} )}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L}{A_H} - \frac{E_L}{E_M} \right)} > 0, \]

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\[
\frac{\partial \omega^{EPE,IC}}{\partial \pi} = (C_H - C_L - (A_L - A_H)) \frac{\omega^{EPE,IC} + (1 - \omega^{EPE,IC}) F \left( \frac{E_L}{E_M} \right)}{(1 - \pi)(C_H - C_L - (A_L - A_H)) + F \left( \frac{A_L - E_L}{A_H - E_M} \right)}
\]

\[= F \times (C_H - C_L - (A_L - A_H))
\times \left( \frac{A_L - E_L}{A_H - E_M} + (1 - \pi)(C_H - C_L - (A_L - A_H)) \left( \frac{E_L}{E_M} \right) \right)
\times \left[ (1 - \pi)(C_H - C_L - (A_L - A_H)) + F \left( \frac{A_L - E_L}{A_H - E_M} \right) \right]^2 > 0.

The sign of \( \frac{\partial \omega^{EPE,IC}}{\partial \pi} \) arises because \( A_L > 1 \) and \( E_L + \left( F \right) \left( 1 - \frac{E_L}{E_M} \right) < \frac{E_L}{E_M} + (1 - \frac{E_L}{E_M}) = 1.\)

As with the ND condition, the effect of \( C_H \) on \( \omega^{EPE,IC} \) is ambiguous. Increasing \( C_H \) increases the stock price from deviating more than the stock price from pooling (as the former effect is one-for-one, but the latter effect is \( \pi \)-for-one), which increases the incentive to deviate and reduces \( \omega^{EPE,IC} \). However, it also increases the capital gain from pooling, which reduces the incentive to deviate and tends to increase \( \omega^{EPE,IC} \). For similar reasons to the above, the effect of \( C_L \) is ambiguous.\(^9\)

The lower bound \( \omega^{EPE,IC} \) is increasing in \( A_L - A_H \). Higher \( A_L - A_H \) augments the capital loss \( L \) suffers if he deviates, which requires a lower weight on fundamental value to encourage deviation. It also reduces the stock price benefit from deviating and being revealed as \( H \), which requires a higher weight on the stock price to encourage deviation. Both forces augment the required \( \omega \). Hence, a rise in \( \omega \) is needed to weaken this fundamental motive not to deviate. The bound is also increasing in \( F \). If \( L \) deviates to selling assets, he suffers a capital loss which is increasing in the amount of assets sold and thus \( F \). Finally, the bound is increasing in \( \pi \): a rise in \( \pi \) reduces \( L \)'s gain in market value from being revealed as type \( H \), so a higher weight on market value is needed for \( L \) to deviate (and thus the IC to be satisfied).

**Proof of Proposition 2**

We need to show that \( \omega^{APE,ND} < \omega^{SE} < \omega^{EPE,IC} \). First, note that \( \omega^{APE,ND} < \omega^{SE} \).

\(^9\)More precisely, the lower bound \( \omega^{EPE,IC} \) is increasing in \( C_H \) if and only if

\[\omega^{EPE,IC}(1 - \pi) < (1 - \omega^{EPE,IC}) \pi F \frac{C_L + A_L + F}{E[C + A] + F}^2,\]

where the LHS captures the stock price effect and the RHS captures the capital loss effect. The lower bound \( \omega^{EPE,IC} \) is increasing in \( -C_L \) if and only if

\[\omega^{EPE,IC}(1 - \pi) < (1 - \omega^{EPE,IC}) \pi F \frac{A_H + C_H + F}{E[C + A] + F}^2.\]
Both expressions are in the form \( M/(M + N) \) which is increasing in \( M \), and this value is greater for \( \omega^{SE} \).

Next, we have \( \omega^{SE} < \omega^{EPE,IC} \) if and only if:

\[
(1 - \pi) \frac{A_L - A_H}{A_H} < \frac{A_L}{A_H} - \frac{C_L + A_L + F}{E[C + A] + F}
\]

\[
\frac{C_L + A_L + F}{E[C + A] + F} < \frac{A_L}{A_H} - (1 - \pi) \frac{A_L - A_H}{A_H} = \frac{\pi A_L + (1 - \pi)A_H}{A_H}
\]

The LHS is less than 1 from (1). The RHS is greater than 1, because it is a weighted average of the denominator and a higher value. Thus, \( \omega^{SE} < \omega^{EPE,IC} \) always holds.

**Proof of Lemma 6**

We must verify that the IC condition (15) is stronger than the ND condition (17). This is true if and only if:

\[
(E^2[A] - A_H A_L) \times (C_L + A_L)(1 + r_H) < (E^2[A] - A_H A_L) \times (C_H + A_H)(1 + r_L)
\]

which always holds since \( \pi \geq \frac{1}{2} \) and \( \frac{1 + r_H}{1 + r_L} < \frac{C_H + A_H}{C_L + A_L} \).

**Proof of Lemma 7**

We start with the ND condition. By pooling, type \( H \)'s fundamental value is

\[
C_H + A_H + R_H - F \left( \frac{C_H + A_H + R_H}{E[C + A + R]} \right).
\]

By deviating, it becomes:

\[
C_H + A_H + R_H - F \left( \frac{A_H}{A_L} \right).
\]

Thus, he will not deviate if:

\[
F \left[ A_H(1 + E[r_H]) - A_L(1 + r_H) \right] \geq A_L(C_H + A_H) - A_H E[C + A]
\]

where

\[
E[r_H] = \pi r_H + (1 - \pi) r_L.
\]
We now move to the IC condition. By pooling, type $L$'s fundamental value is

$$C_L + A_L + R_L - F \left( \frac{C_L + A_L + R_L}{E[C + A + R]} \right).$$

By deviating to asset sales and being inferred as type $H$, it becomes:

$$C_L + A_L + R_L - F \left( \frac{A_L}{A_H} \right).$$

Thus, he will deviate if:

$$F [A_H(1 + r_L) - A_L (1 + E[r])] \geq A_L E[C + A] - A_H(C_L + A_L)$$

The IC condition is stronger if and only if:

$$(\pi A_H^2 + (1 - \pi)A_L^2) \times [(1 + r_L)(C_H + A_H) - (1 + r_H)(C_L + A_L)] > A_H A_L \times [(1 + r_L)(C_H + A_H) - (1 + r_H)(C_L + A_L)]$$

which always holds since $\pi \geq \frac{1}{2}$ and $\frac{1 + r_H}{1 + r_L} < \frac{C_H + A_H}{C_L + A_L}$.

**Proof of Lemma 8**

As in the core model, it is automatic that $L$ will not deviate. Following similar steps to the core model, $H$ will not deviate if:

$$\omega \geq \frac{F \left( \frac{C_H + A_H + F(1 + r_H)}{E[C + A] + F(1 + E[r])} - \frac{A_H}{A_L} \right)}{\pi ((C_H - C_L) - (A_L - A_H)) + F \left( \frac{C_H + A_H + F(1 + r_H)}{E[C + A] + F(1 + E[r])} - \frac{A_H}{A_L} \right)}$$

and the IC condition is satisfied if:

$$\omega \geq \frac{F \left( \frac{A_L}{A_H} - \frac{C_L + A_L + F(1 + r_L)}{E[C + A] + F(1 + E[r])} \right)}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L}{A_H} - \frac{C_L + A_L + F(1 + r_L)}{E[C + A] + F(1 + E[r])} \right)}$$

Using similar steps to the proof of Proposition 5, the IC condition is stronger than the ND condition.

**Proof of Proposition 3**
We only need to show that $F_{APE,IC,I} > F_{EPE,IC,I}$. This holds if:

$$(1 + r_L)E[A]E[C + A] + (1 + r_H)A_H(C_L + A_L) + (1 + E[r_0])A_L(C_H + A_H)$$

$$< (1 + r_L)A_H(C_H + A_H) + (1 + r_H)A_LE[C + A] + (1 + E[r_0])E[A](C_L + A_L)$$

If we set $r_H = 0$, this inequality will be satisfied. If we set $r_H = r_L$, it will still be satisfied: the inequality becomes


$$< A_H(C_H + A_H) + A_LE[C + A] + E[A](C_L + A_L)$$

or equivalently


$$\pi E[C + A] + (1 - \pi)(C_L + A_L) < C_H + A_H$$

which always holds. Returning to the original inequality, the derivative with respect to $r_H$ is greater on the LHS than on the RHS: The derivatives are, for the LHS,

$$A_H(C_L + A_L) + \pi A_L(C_H + A_H)$$

and for the RHS,

$$A_L E[C + A] + \pi E[A](C_L + A_L).$$

which is greater. Thus, starting at any pair $r_H = r_L$, we can increase $r_H$ sufficiently until the bounds coincide. This occurs at $\frac{1+r_H}{1+r_L} = \frac{C_H + A_H}{C_L + A_L}$. However, since $1 + \frac{r_H}{1+r_L} < \frac{C_H + A_H}{C_L + A_L}$ from equation (14), these bounds never coincide and so we always have $F_{APE,IC,I} > F_{EPE,IC,I}$.

If $\frac{1+E[r_0]}{1+r_L} > \frac{A_H}{A_L}$, EPE not sustainable for any $F$, that is $F_{EPE,IC,I} = \infty$. Since $A_H > E[A]$ and $1 + r_H > 1 + E[r_0]$, in this case we will also have $\frac{1+r_H}{1+r_L} > \frac{E[A]}{A_L}$. Then APE will be sustainable for any $F$, that is $F_{APE,IC,I} = \infty$. Thus even in the boundary case, $F_{EPE,IC,I}$ is no greater than $F_{APE,IC,I}$.

**Proof of Proposition 4**

Since $\omega_{APE,ND}$ and $\omega_{SE}$ are unchanged from the core model, we still have $\omega_{APE,ND} <$
We need to check that \( \omega^{SE} < \omega^{EPE,IC,I} \). To do so, rewrite \( \omega^{SE} \) as

\[
\omega^{SE} = \frac{F \left[ (1 - \pi) \left( \frac{A_L}{A_H} - 1 \right) \right]}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F \left[ (1 - \pi) \left( \frac{A_L}{A_H} - 1 \right) \right]}.
\]

Note that \( \omega^{SE} \) is identical to \( \omega^{EPE,IC,I} \) except for the bracketed terms. The expressions are increasing in those bracketed terms, so it will be sufficient to show that

\[
(1 - \pi) \left( \frac{A_L}{A_H} - 1 \right) < \frac{A_L}{A_H} - \frac{C_L + A_L + F(1 + r_L)}{E[C + A] + F(1 + E[r_H])}.
\]

This automatically holds since low equity is less valuable than average equity, \( C_L + A_L + F(1 + r_L) < E[C + A] + F(1 + E[r_H]) \).

### B Finishing from Multiple Sources

The core model assumes that firms can only raise financing from a single source. One potential justification is that the transactions costs from using multiple sources of financing are prohibitive. In addition, the assumption can be endogenized by the off-equilibrium-path belief that any firm that issues multiple financing sources is of type \( L \). This section studies whether this belief satisfies the IC. We show that, for three of the four pooling equilibria studied in the paper, our existing IC condition is sufficient to rule out this deviation. However, for the negative-correlation APE (in which there is currently no IC condition as it is trivial), we have to impose a new condition (20) to ensure that the required belief satisfies the IC.

We wish to show that type \( L \) would deviate to any available mix of assets and equity if he were inferred to be a high type from doing so. That is, if type \( L \) raises \( \alpha F \) of funds from asset sales and \( (1 - \alpha) F \) from equity issuance, and is inferred as type \( H \), his payoff is higher than in the pooling equilibrium for all \( \alpha \). We consider the four pooling equilibria in turn.

**Positive correlation, asset-pooling.** If we impose our existing IC condition (6), \( L \)'s capital gain from selling equity at a high price exceeds his gain from selling assets at a pooled price, so he deviates:

\[
\frac{C_L + A_L + F}{C_H + A_H + F} \leq \frac{A_L}{\pi A_H + (1 - \pi)A_L}.
\]
We wish to show that his gain from selling any mix of assets and equity at a high price exceeds his gain from selling assets at a pooled price:

\[
\alpha \frac{A_L}{A_H} + (1 - \alpha) \frac{C_L + A_L + F}{C_H + A_H + F} \leq \frac{A_L}{\pi A_H + (1 - \pi) A_L}
\]

The existing IC condition (6) establishes the inequality for \( \alpha = 0 \). Moreover, the LHS is linear in \( \alpha \), and for \( \alpha = 1 \) it simplifies to \( \frac{A_L}{A_H} \leq \frac{A_L}{E[A]} \), which holds because positive correlation implies \( A_H > E[A] \). Thus, the inequality is satisfied for all \( \alpha \in [0, 1] \).

**Positive correlation, equity-pooling.** We wish to show that, for all \( \alpha \):

\[
\alpha \frac{A_L}{A_H} + (1 - \alpha) \frac{C_L + A_L + F}{C_H + A_H + F} \leq \frac{C_L + A_L + F}{E[C + A] + F}
\]

The LHS is again linear in \( \alpha \). The IC condition (8) in the core model establishes the inequality for \( \alpha = 1 \), and for \( \alpha = 0 \) the LHS simplifies to

\[
\frac{C_L + A_L + F}{C_H + A_H + F} \leq \frac{C_L + A_L + F}{E[C + A] + F}
\]

which follows from high equity being more valuable than low equity.

**Negative correlation, asset-pooling.** We wish to show that, for all \( \alpha \):

\[
\omega E[C + A] + (1 - \omega) \left( C_L + A_L + F - F \left( \frac{A_L}{E[A]} \right) \right) \\
\leq \omega (C_H + A_H) + (1 - \omega) \left( C_L + A_L + F - F \left( \frac{\alpha A_L}{A_H} + (1 - \alpha) \frac{C_L + A_L + F}{C_H + A_H + F} \right) \right)
\]

We can rearrange this condition to \( \omega \geq \frac{MF}{\kappa + MF} \), where \( \kappa \equiv (C_H - C_L) - (A_L - A_H) \) and \( M \) is defined appropriately. For \( \alpha = 0, M < 0 \) and the lower bound on \( \omega \) is negative, which is why the IC condition was trivial in the core model. However, the derivative of the bound with respect to \( \alpha \) is positive: it equals the sign of \( \frac{\partial M}{\partial \alpha} \), which is positive since \( A_L > A_H \) under negative correlation. Thus, as \( \alpha \) increases, the lower bound on \( \omega \) rises and can eventually become positive and constitute a nontrivial IC condition.

Intuitively, if type \( L \) is inferred as type \( H \), his capital loss to selling assets is even higher than under the pooling equilibrium, since he receives a low price \( A_H \) rather than a pooled price. Thus, financing mixes that are primarily comprised of assets (high \( \alpha \)) are particularly costly to him and he may be unwilling to deviate even if he is inferred as type \( H \). To ensure that \( L \) is willing to deviate for all \( \alpha \), we require the condition to be
satisfied for $\alpha = 1$. This in turn requires:

$$\omega \geq \frac{F\left(\frac{A_L}{A_H} - \frac{A_L}{E[A]}\right)}{(C_H - C_L) - (A_L - A_H) + F\left(\frac{A_L}{A_H} - \frac{A_L}{E[A]}\right)}$$

(20)

To encourage deviation, $L$ must have a high weight on the stock price to offset his capital loss from asset sales. Condition (20) is stronger than (10) if and only if $(1 - \pi)A_L > A_H$, which is not imposed by any of our assumptions thus far. Thus, if $(1 - \pi)A_L > A_H$, the additional condition (20) is needed to formally rule out the use of multiple financing sources.

**Negative correlation, equity pooling.** We wish to show that, for all $\alpha$:

$$\omega E[C + A] + (1 - \omega) \left(C_L + A_L + F - F\left(\frac{C_L + A_L + F}{E[C + A] + F}\right)\right)$$

$$\leq \omega(C_H + A_H) + (1 - \omega) \left(C_L + A_L + F - F\left(\frac{A_L}{A_H} + (1 - \alpha)\frac{C_L + A_L + F}{C_H + A_H + F}\right)\right).$$

The existing IC condition (12) establishes the inequality for $\alpha = 1$. The derivative of the RHS with respect to $\alpha$ is

$$-F(1 - \omega) \left[\frac{A_L}{A_H} - \frac{C_L + A_L + F}{C_H + A_H + F}\right] < 0.$$

Thus, as we decrease $\alpha$ from 1 to 0, the RHS increases and the inequality is *a fortiori* satisfied.